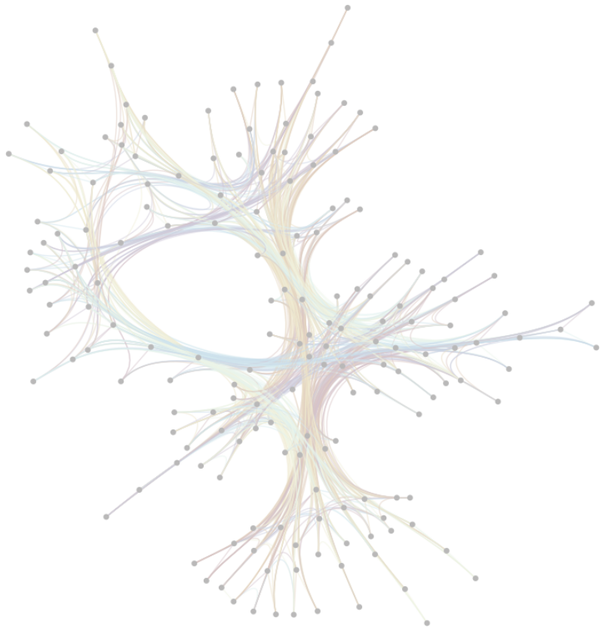
TUM School of Computation, Information and Technology
Technical University of Munich

Bundling-Aware Graph Drawing

GD' 24 · Vienna · 18.09.2024

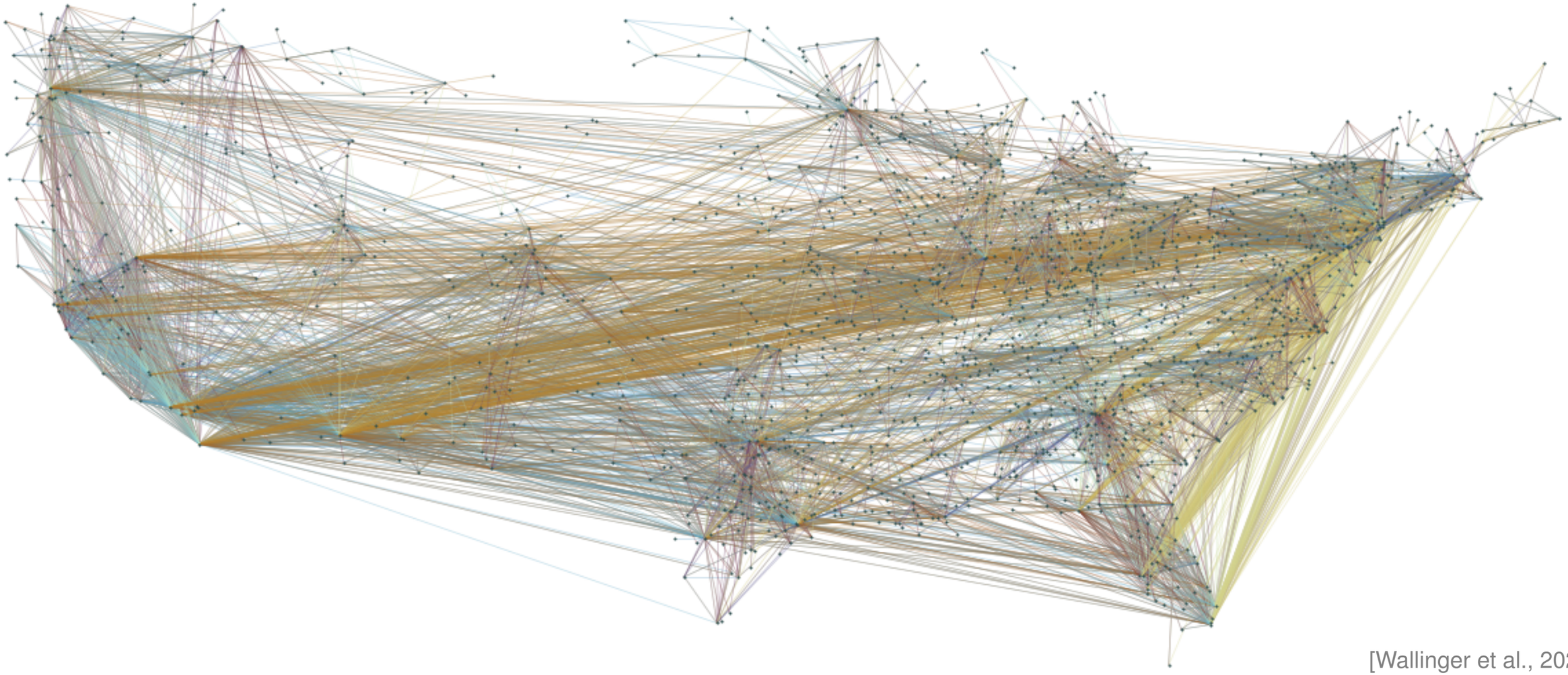
Markus Wallinger



Daniel Archambault
Newcastle University
Tommaso Piselli
Giuseppe Liotta
Alessandra Tappini
Martin Nöllenburg
TU WIEN
Markus Wallinger

Traditional Edge Bundling

- Clutter reduction technique
- Input: graph and drawing $\rightarrow$ **post-processing**



[Wallinger et al., 2023]

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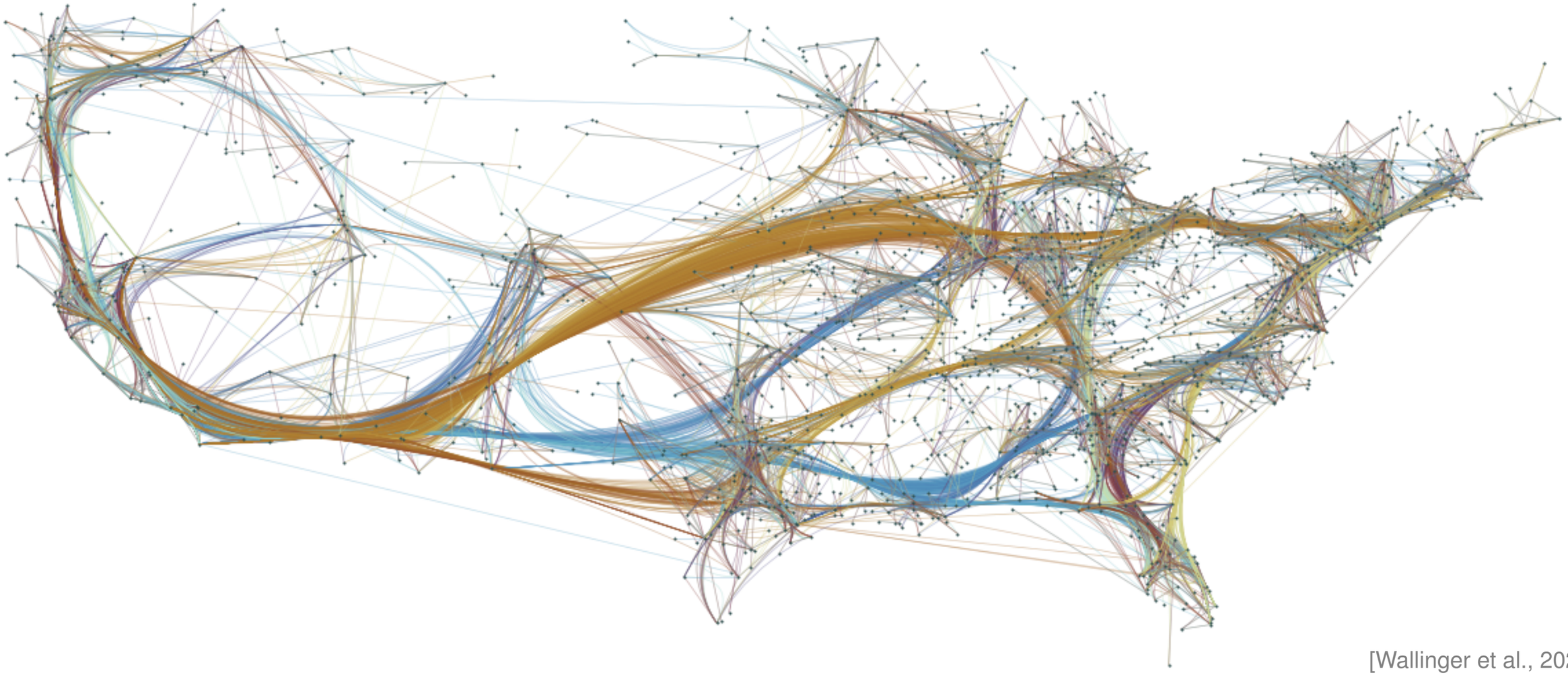
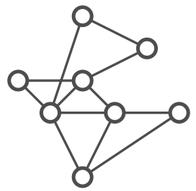


[Wallinger et al., 2023]

Motivation

- What if we **don’t have a drawing?**



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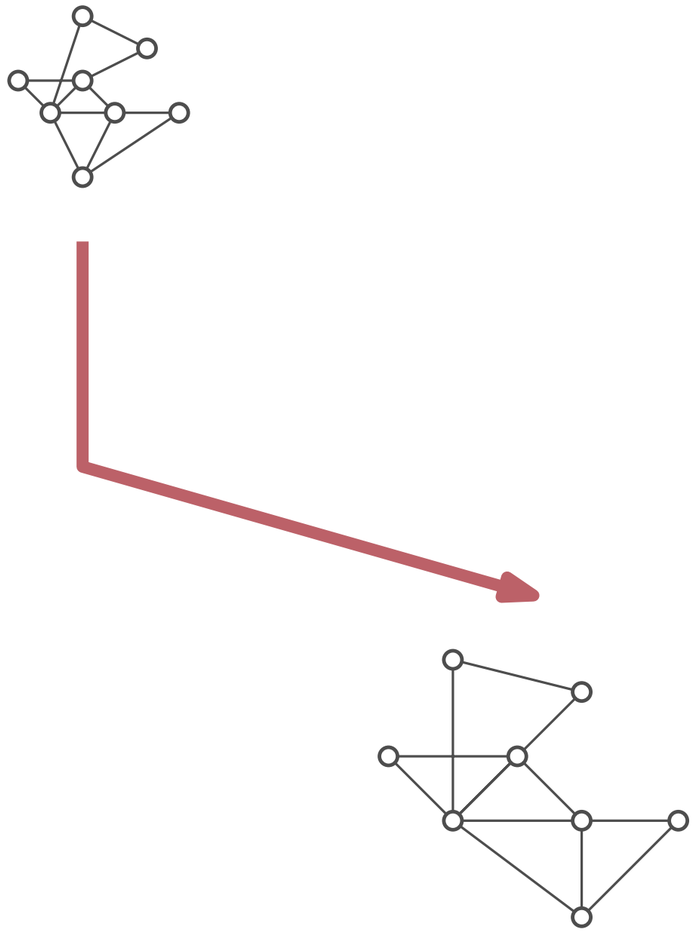

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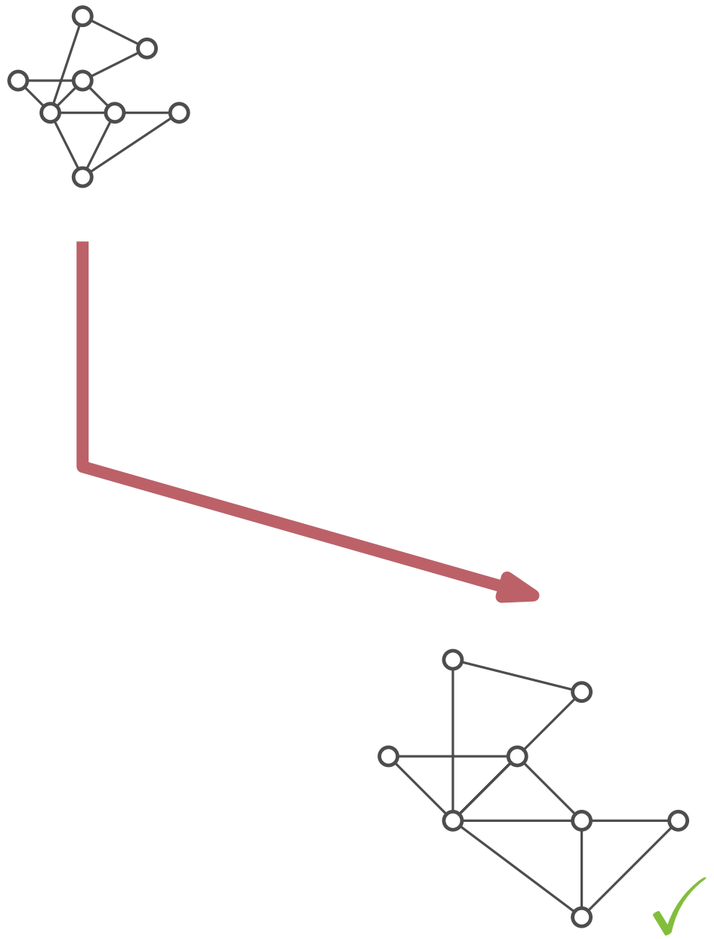

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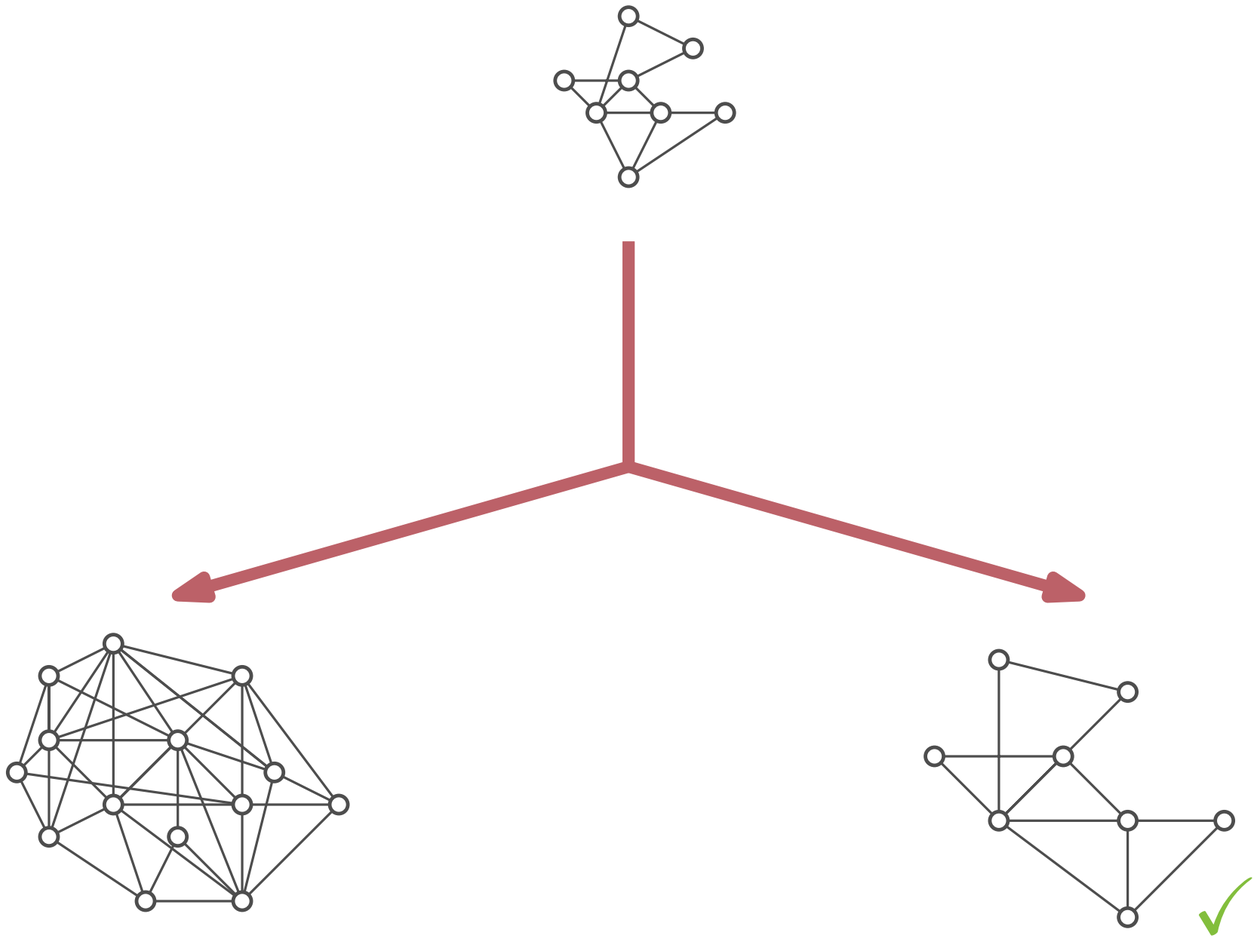

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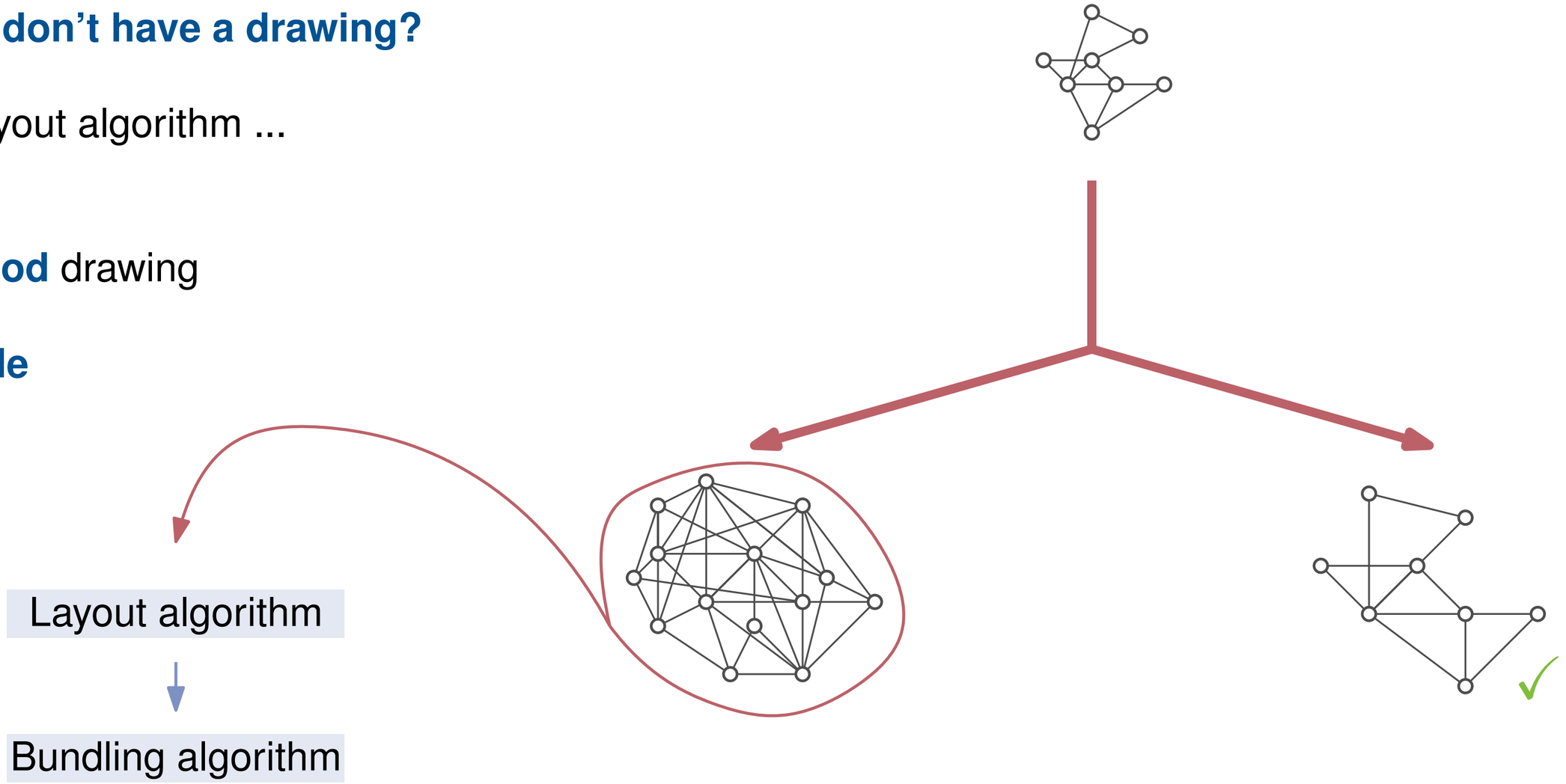

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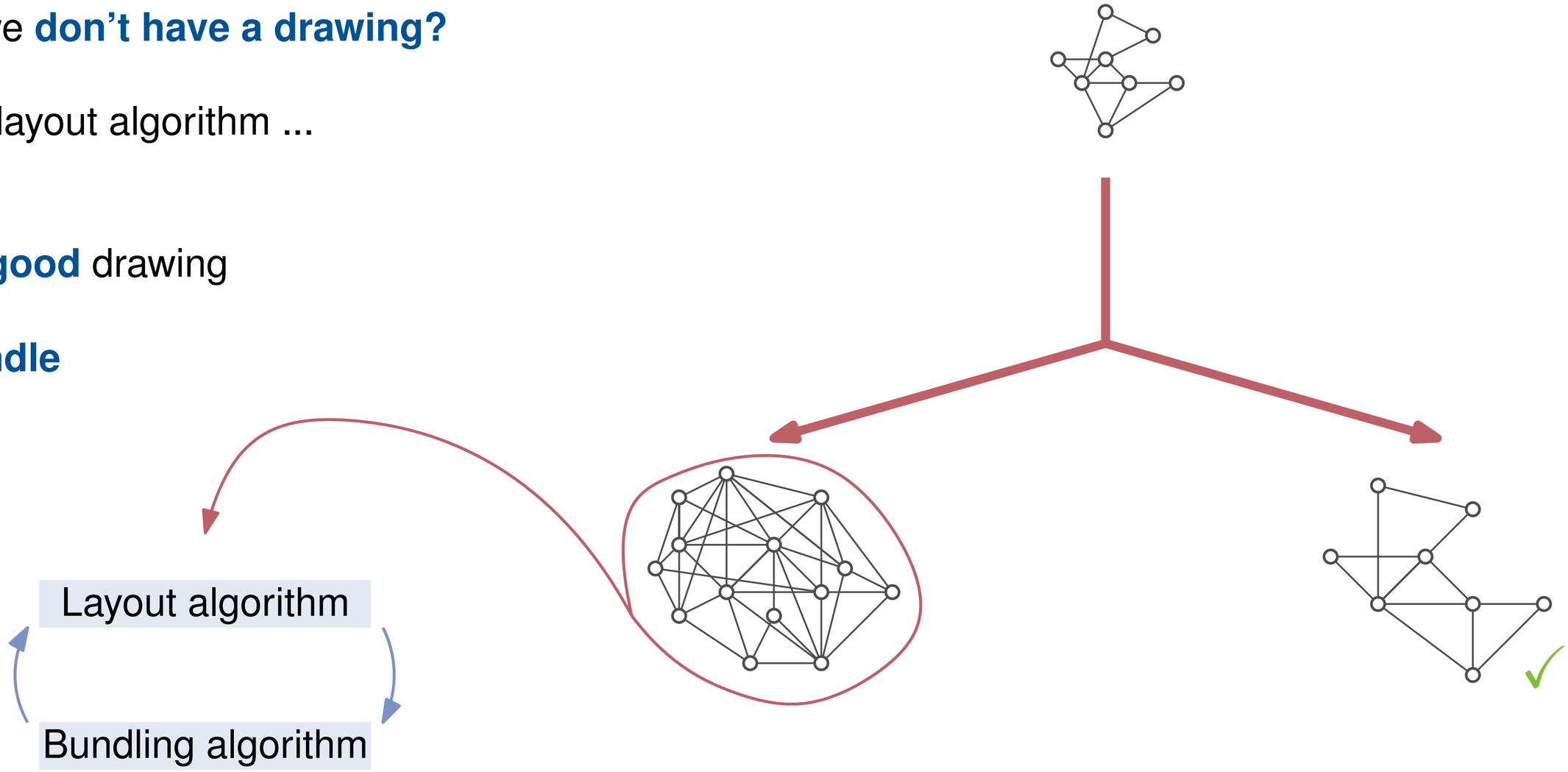

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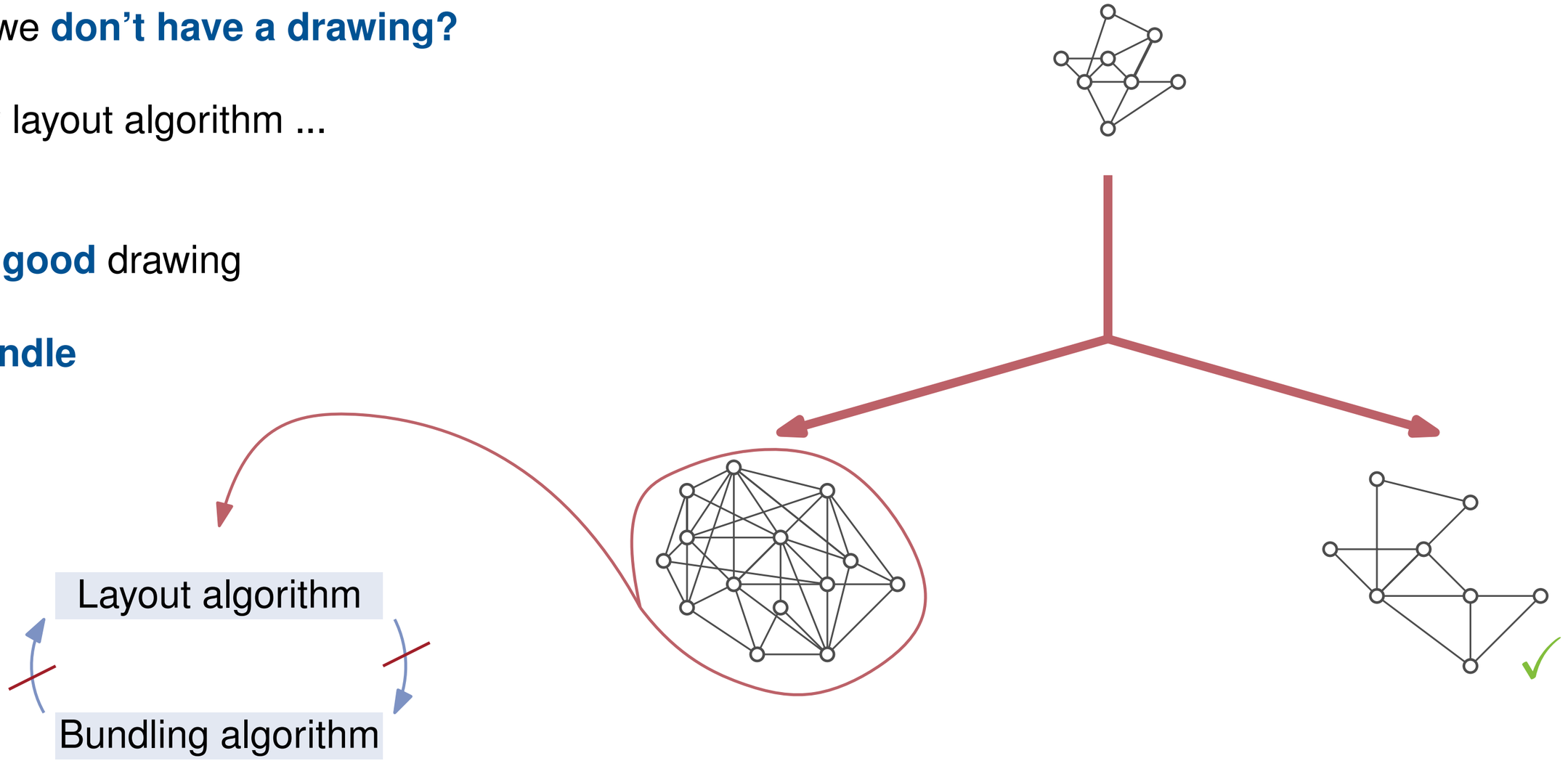

→ **Can we compute a layout that has good properties for bundling?**

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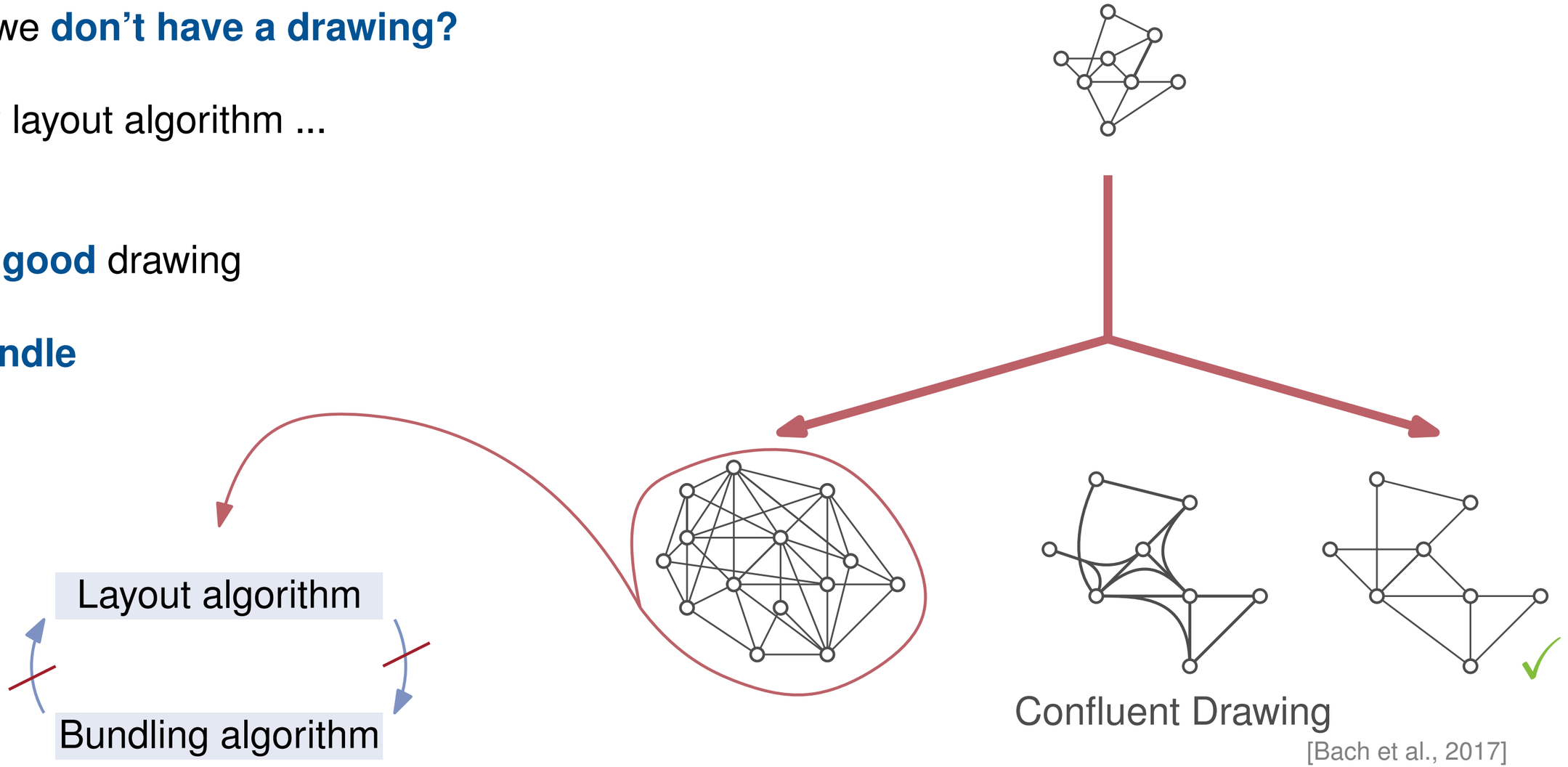

[Bach et al., 2017]

[Zheng et al., 2021]

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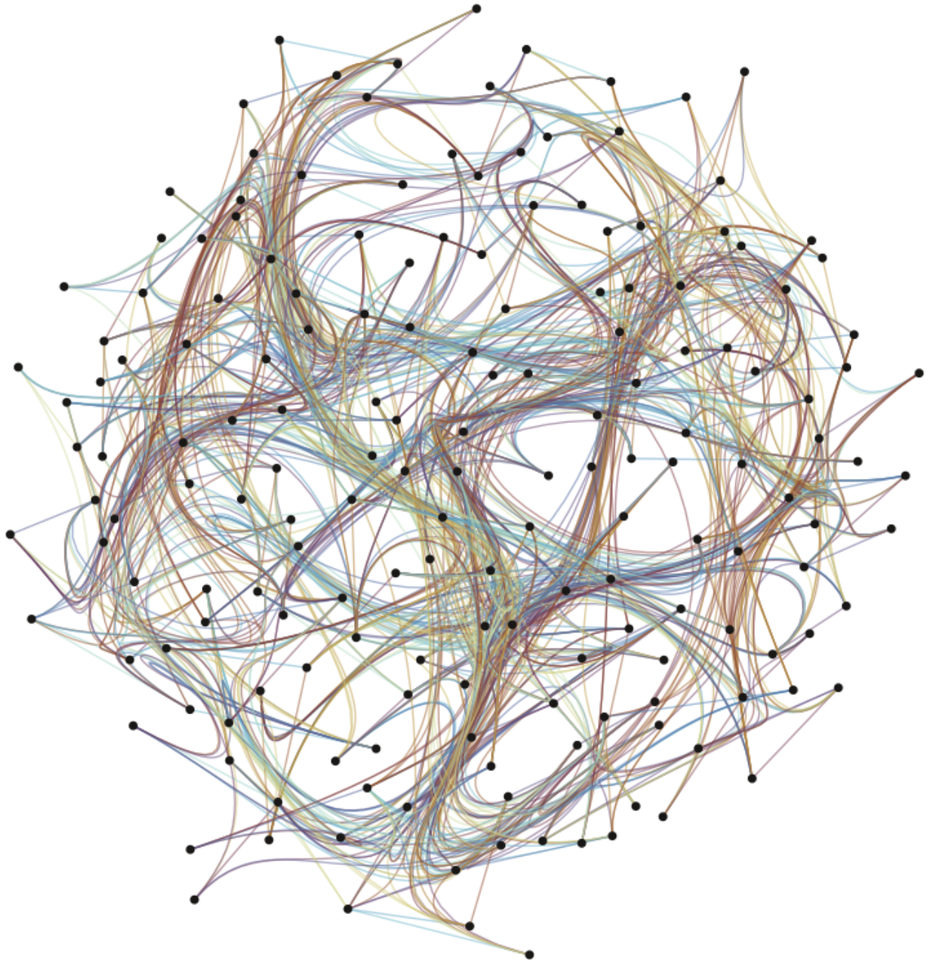
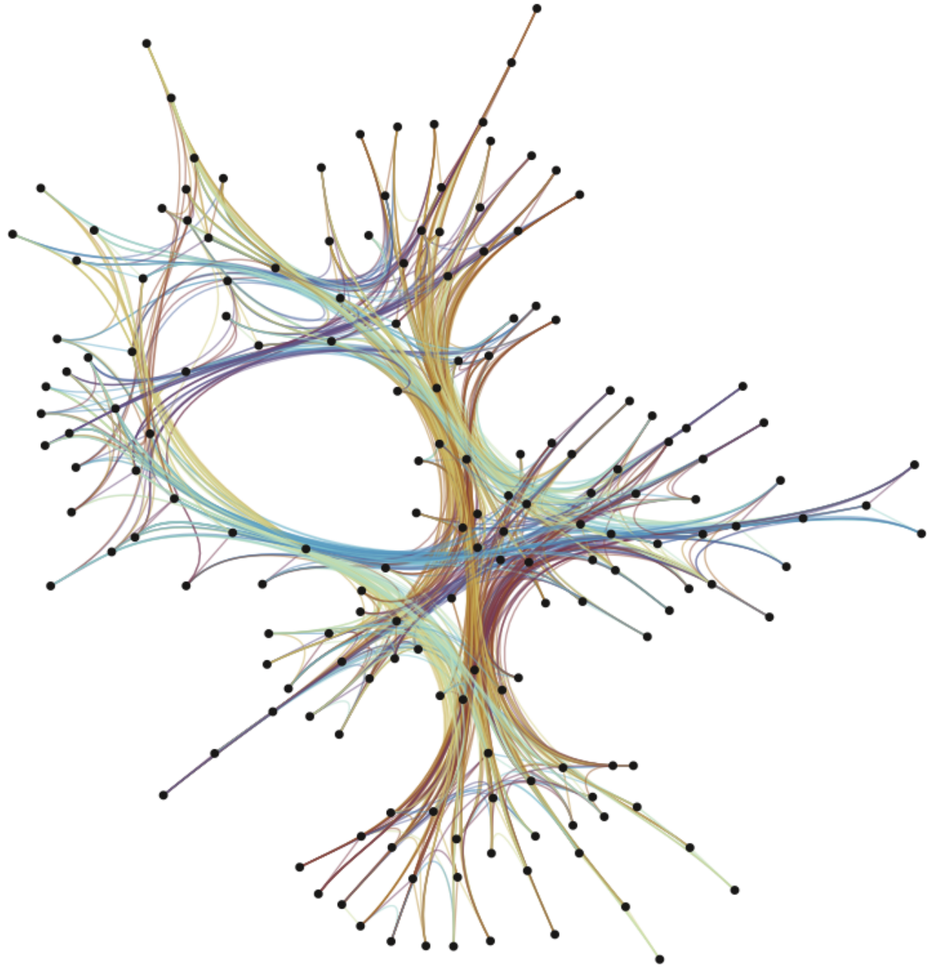


Stress majorization [Zheng et al., 2019]

S-EPB bundling [Wallinger et al., 2023]

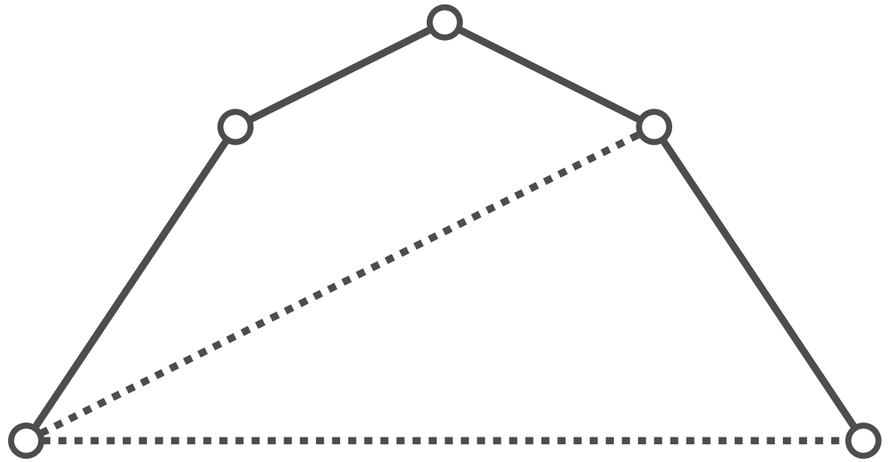


Filter-Draw-Bundle Framework

Reminder: Spanner Edge-Path Bundling (S-EPB)*

- Edge bundling technique for graphs

- Input: **graph** $G = (V, E)$, **drawing** $\Gamma(G)$ and **distortion** parameter t



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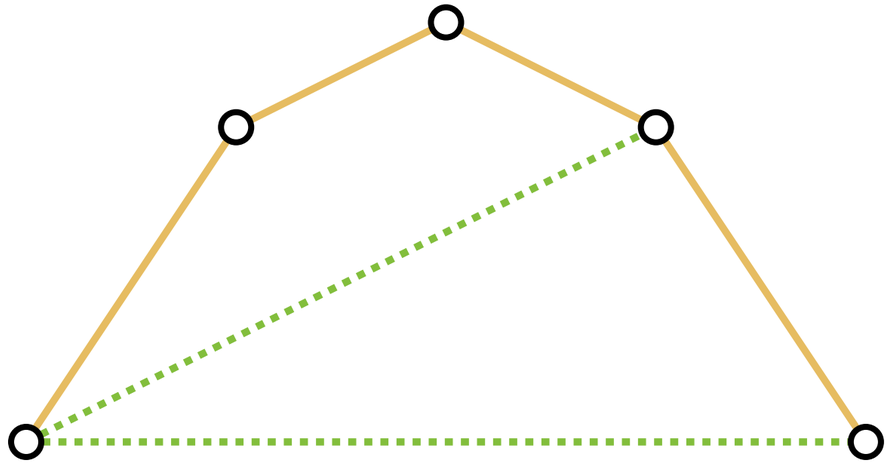
  (Greedy spanner algorithm) [Althöfer et al., 1993]



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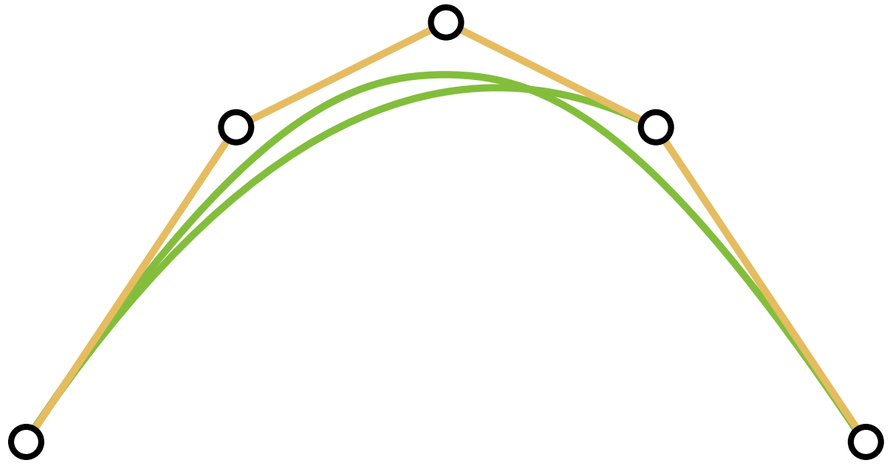

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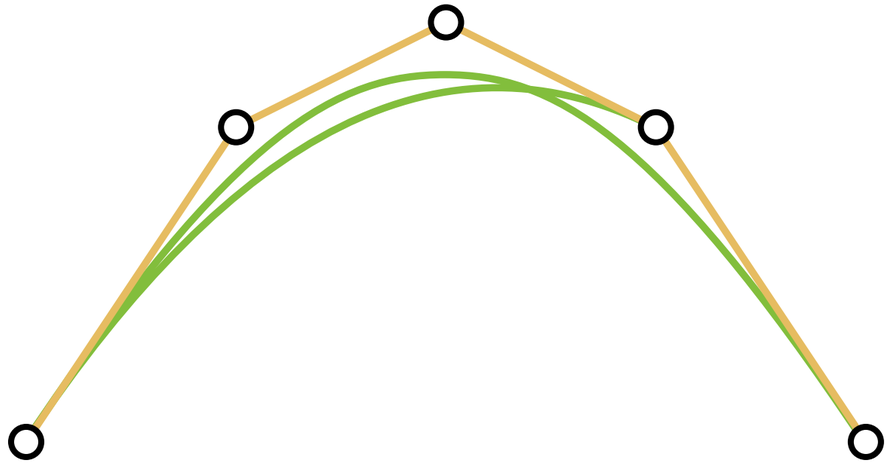

→ **we base our framework on S-EPB**

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Quality Metrics

Ink Ratio

- Ratio of **colored** pixels to **uncolored pixels**

Distortion

Ambiguity

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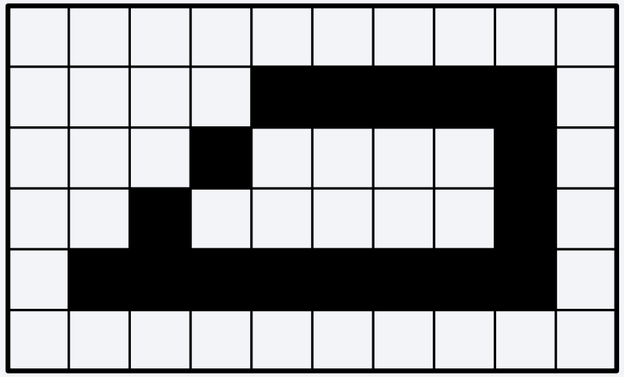

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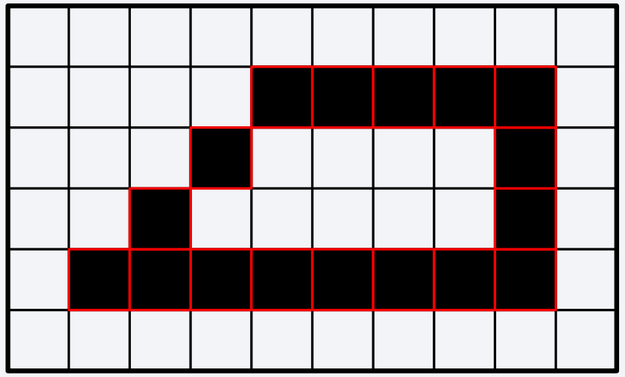

$$\frac{13}{}$$

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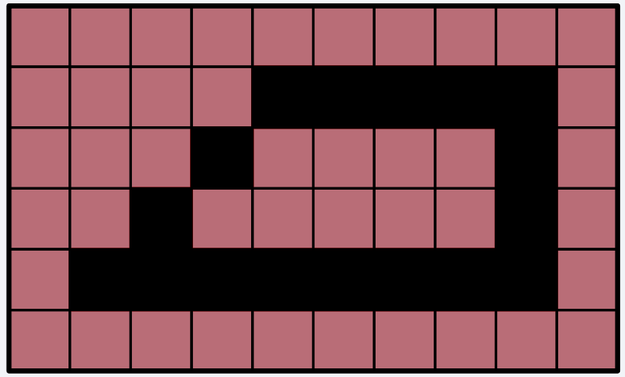

$$\frac{13}{42}$$

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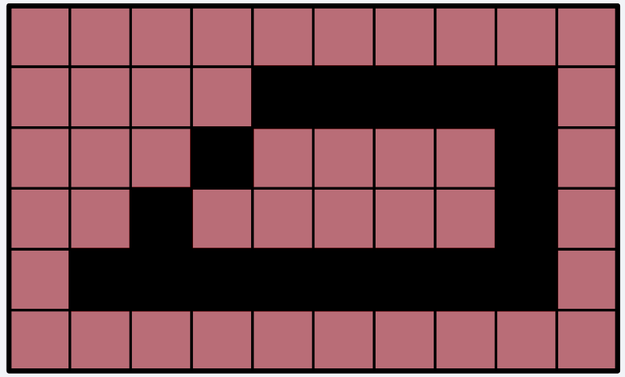
- Lower is better



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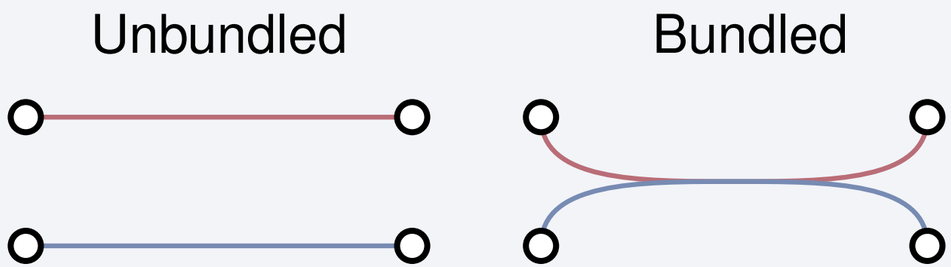

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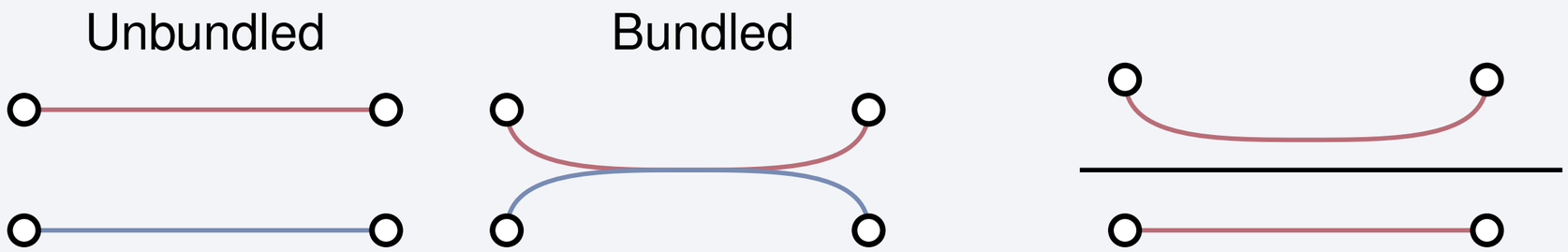

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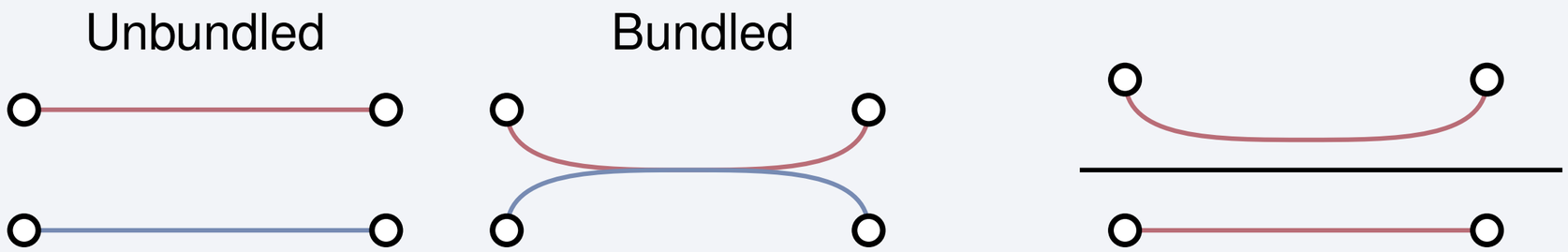

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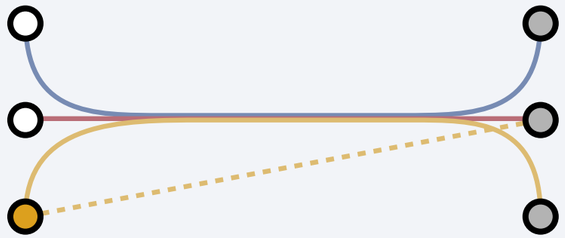
- Metric that tries to capture **faithfulness**
- Parallel edges and shallow crossings

Perceived adjacencies



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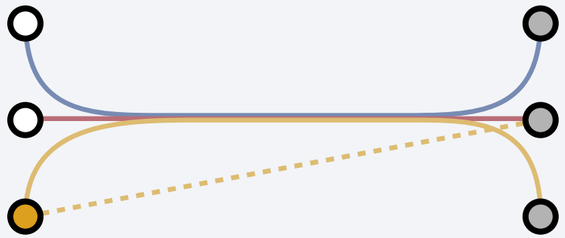

True/False adjacencies

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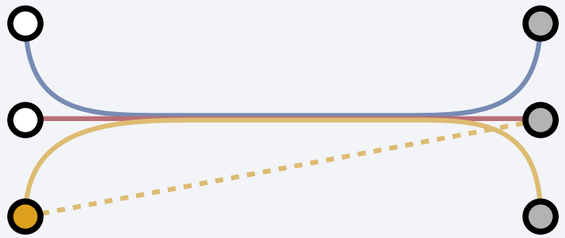

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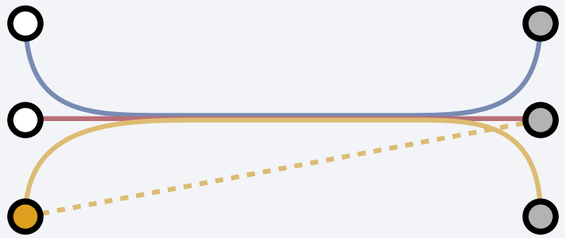

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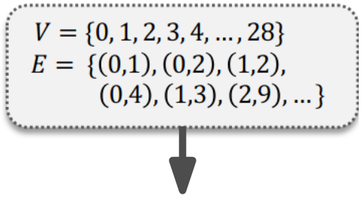

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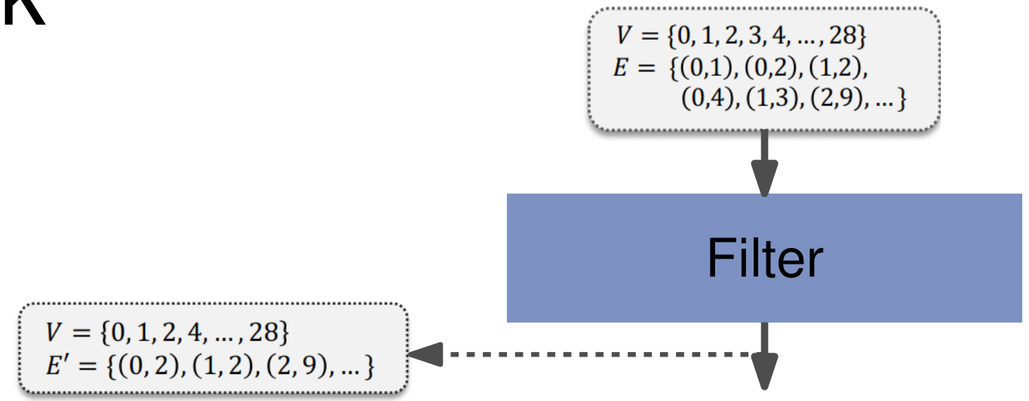

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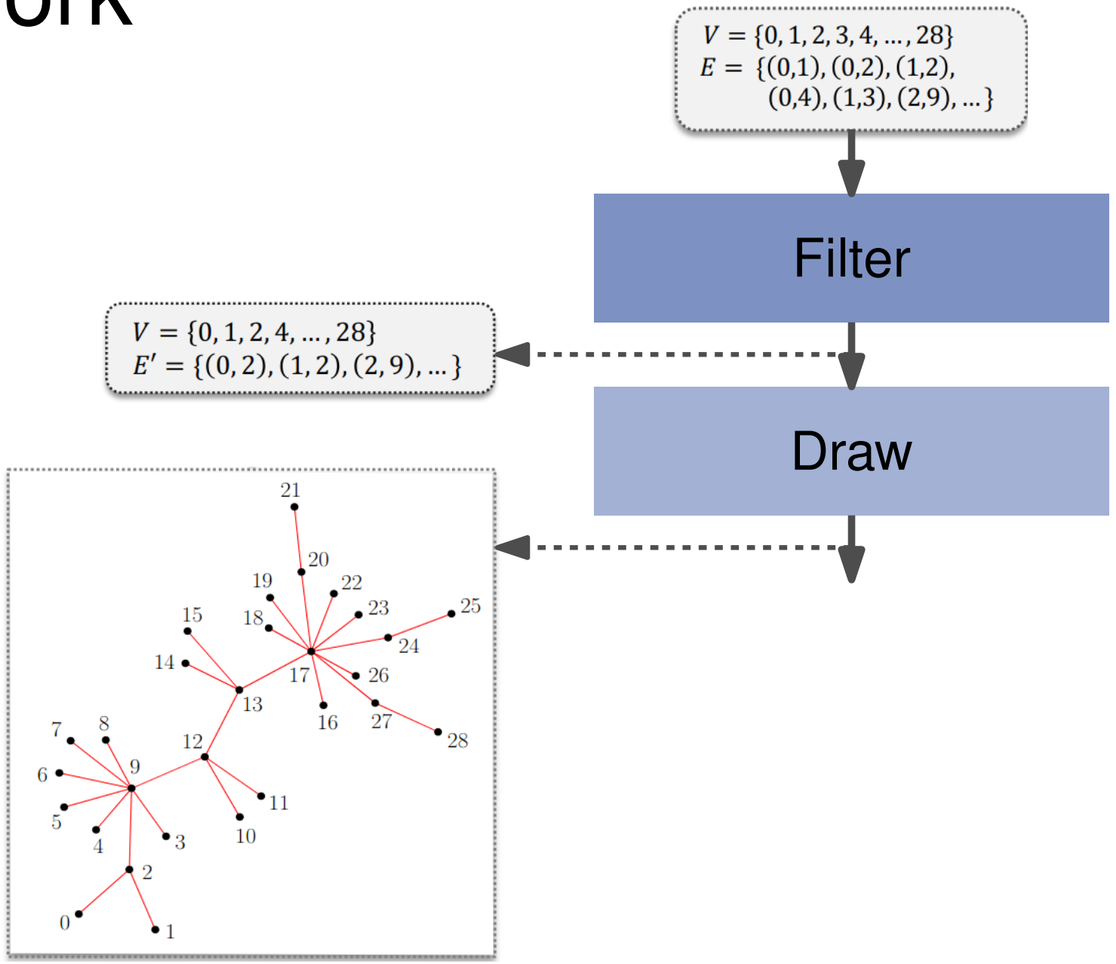

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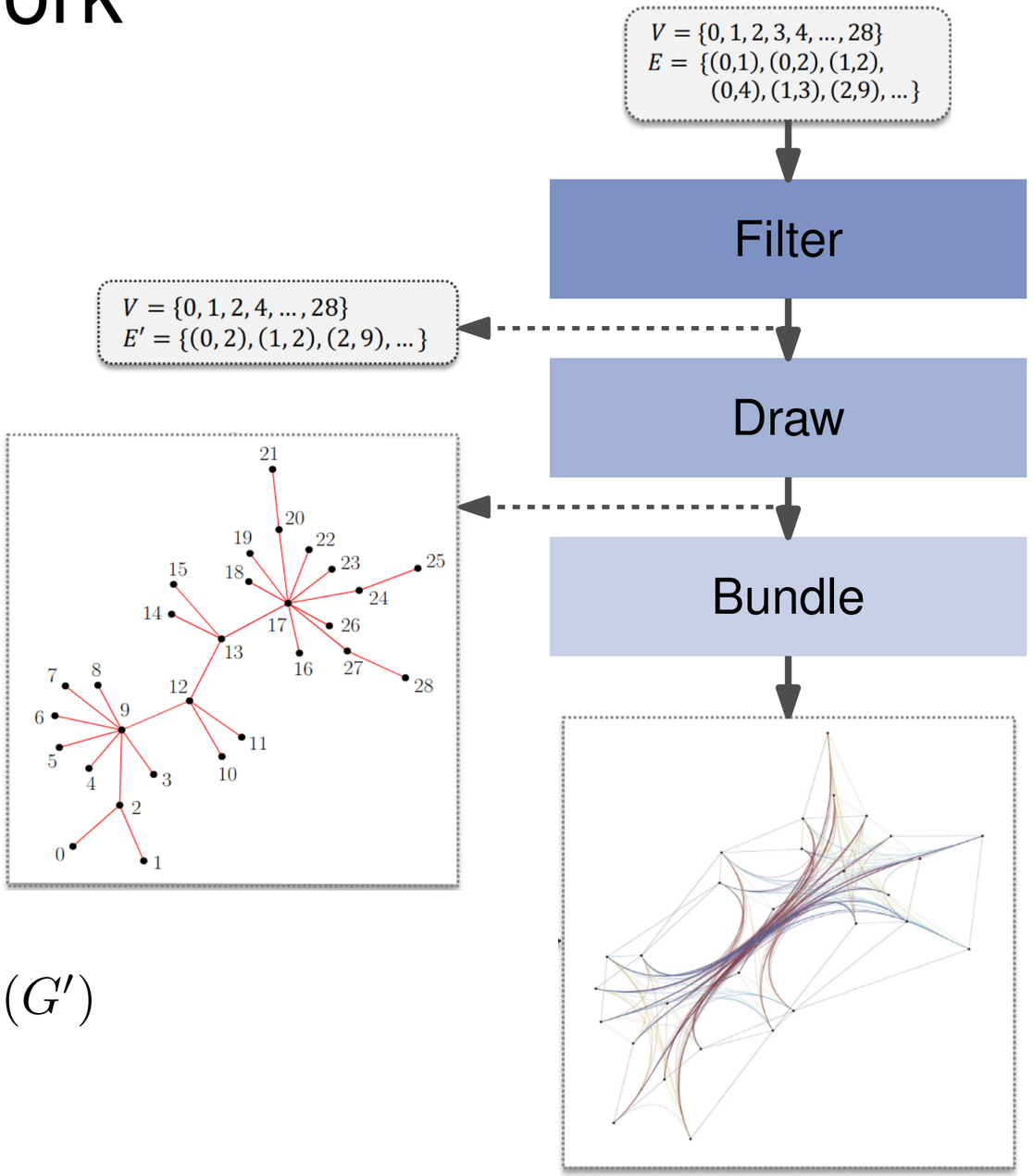

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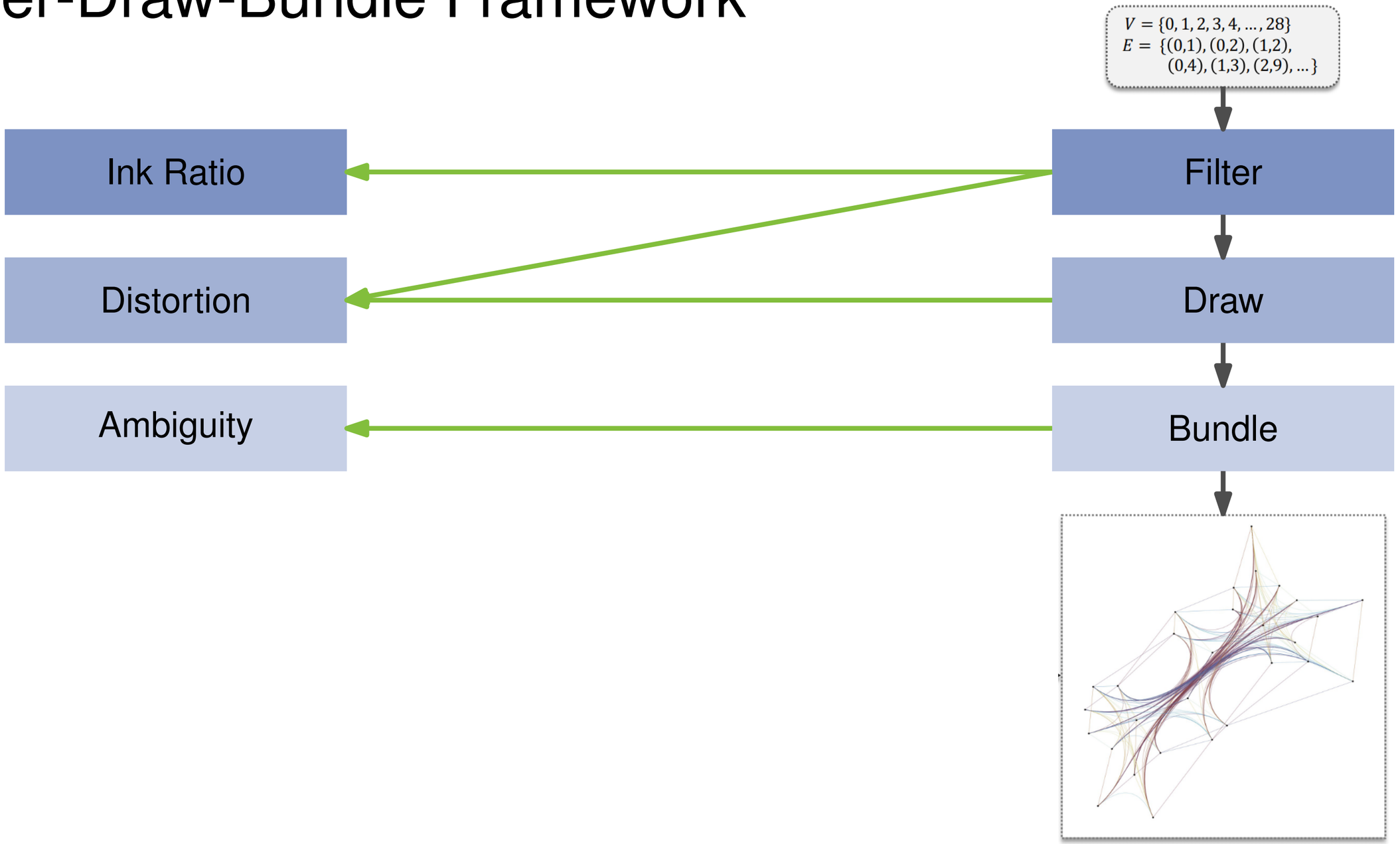

FDB - Edge Weights

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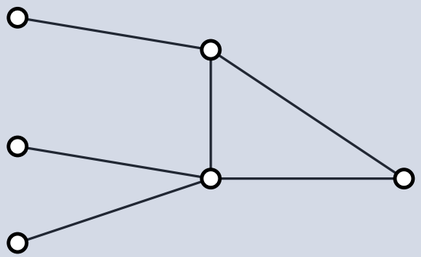
- Traditional edge betweenness - Compute APSP and count how often paths pass through each edge 	

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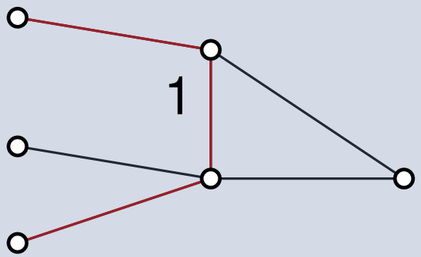

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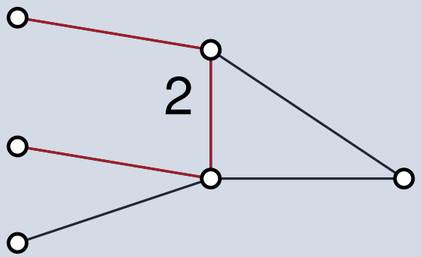

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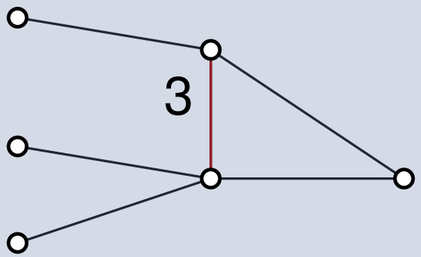

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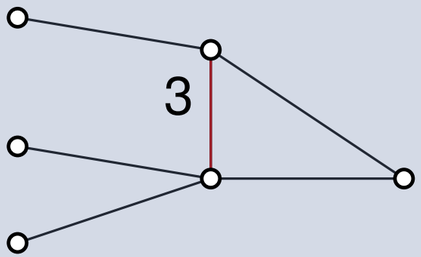

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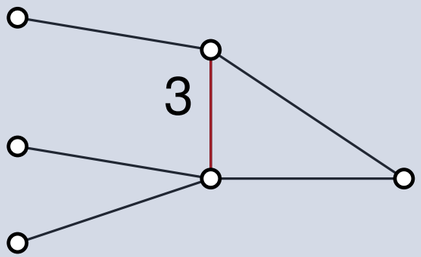
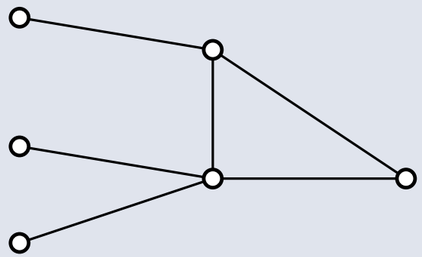
- Traditional edge betweenness - Compute APSP and count how often paths pass through each edge 	- Edge betweenness that only considers adjacent vertices - Remove each edge and compute alternate SP - Count how often paths pass through each edge 

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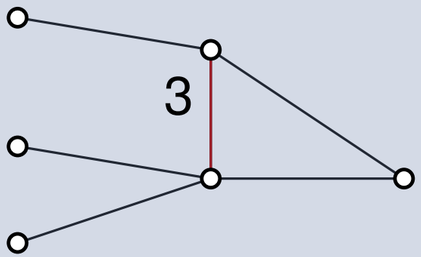

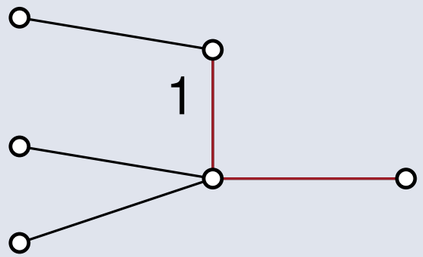

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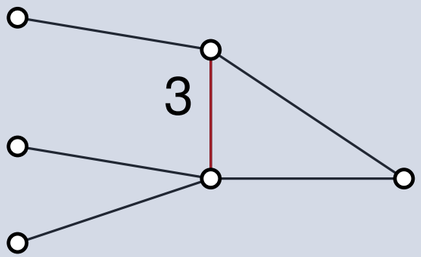
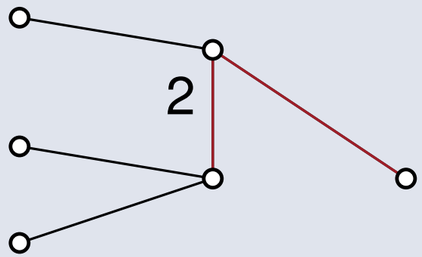
- Traditional edge betweenness - Compute APSP and count how often paths pass through each edge 	- Edge betweenness that only considers adjacent vertices - Remove each edge and compute alternate SP - Count how often paths pass through each edge 

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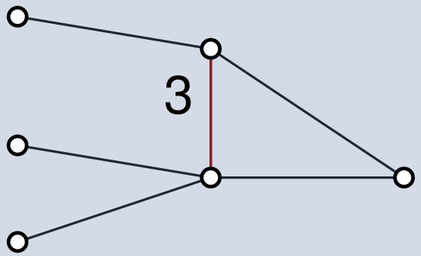
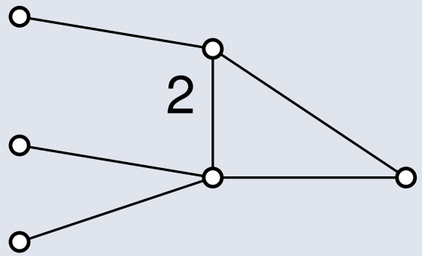
Edge Betweeness	Neighboring Edge Betweeness
Traditional edge betweenness Compute APSP and count how often paths pass through each edge	Edge betweenness that only considers adjacent vertices Remove each edge and compute alternate SP Count how often paths pass through each edge



Lastly, normalize and invert



Filter-Draw-Bundle Framework

1. Filter

   Assign weight to edges

   Compute t-spanner $G' = (V, E' \subseteq E)$

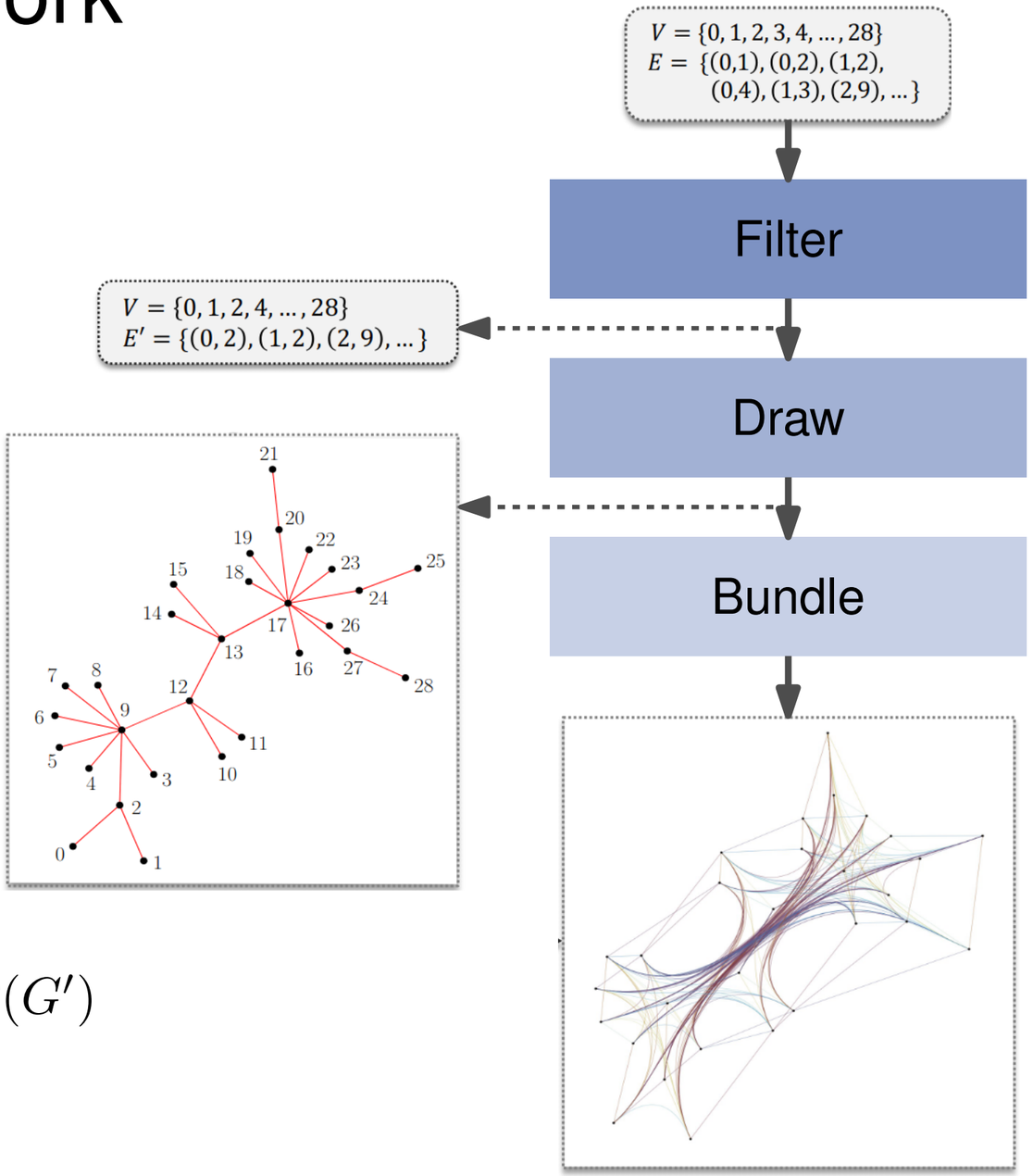
2. Draw

   Compute drawing $\Gamma(G')$

   Stochastic Gradient Descent

3. Bundle

   Bundle remaining edges $E \setminus E'$ against $\Gamma(G')$



Experimental Evaluation - Setup

- Four **graph sizes**: $|V| = \{[20, 50], [50, 100], [100, 150], [150, 200]\}$
- Stochastic Block Model

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- Four algorithms

EB-FDB

Edge betweeness to filter
Stochastic Gradient Descent(SGD) to draw
S-EPB to bundle

NEB-FDB

Neighboring edge betweeness
SGD to draw
S-EPB to bundle

PP-Bundling

SGD to draw
S-EPB to bundle

Confluent [Zheng et al., 2021]

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- Four algorithms

We compute with **distortion** $t = \{2, 4, \ldots, 10\}$

EB-FDB

Edge betweeness to filter
Stochastic Gradient Descent(SGD) to draw
S-EPB to bundle

NEB-FDB

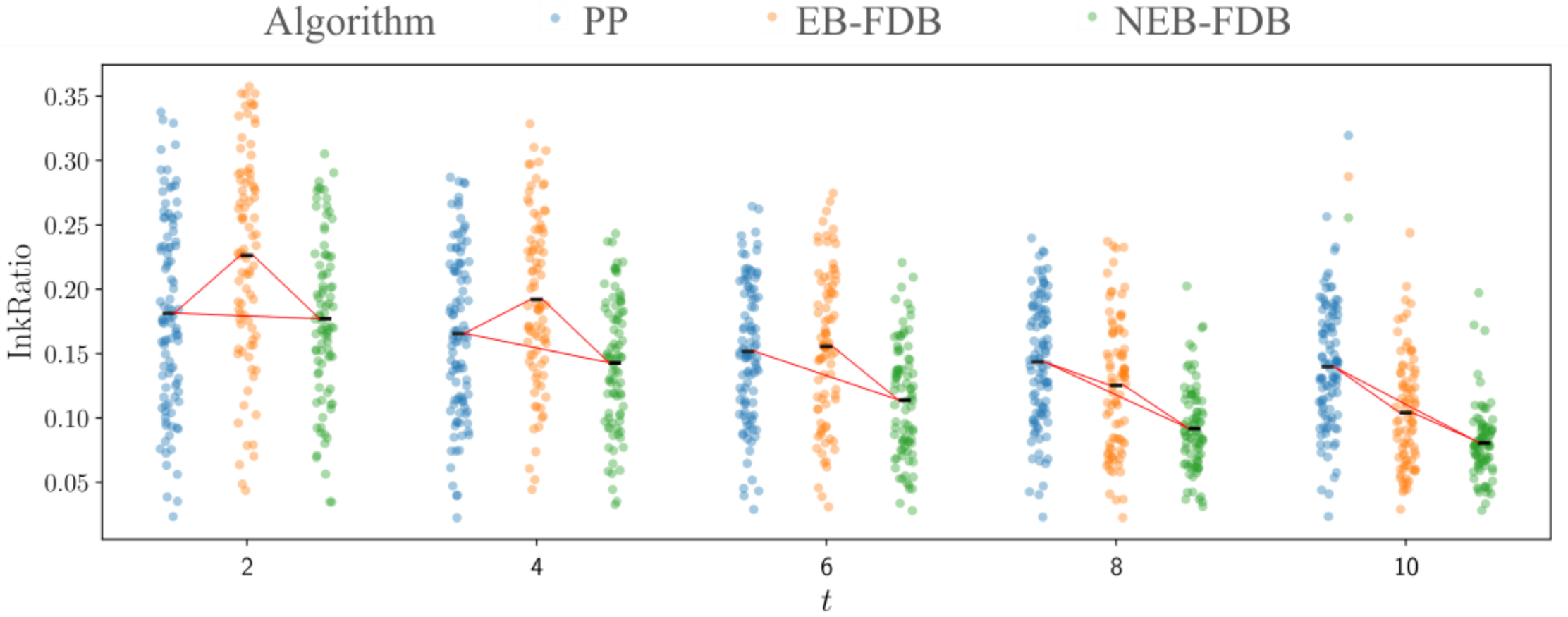
Neighboring edge betweeness
SGD to draw
S-EPB to bundle

PP-Bundling

Confluent [Zheng et al., 2021]

Experimental Evaluation – Results

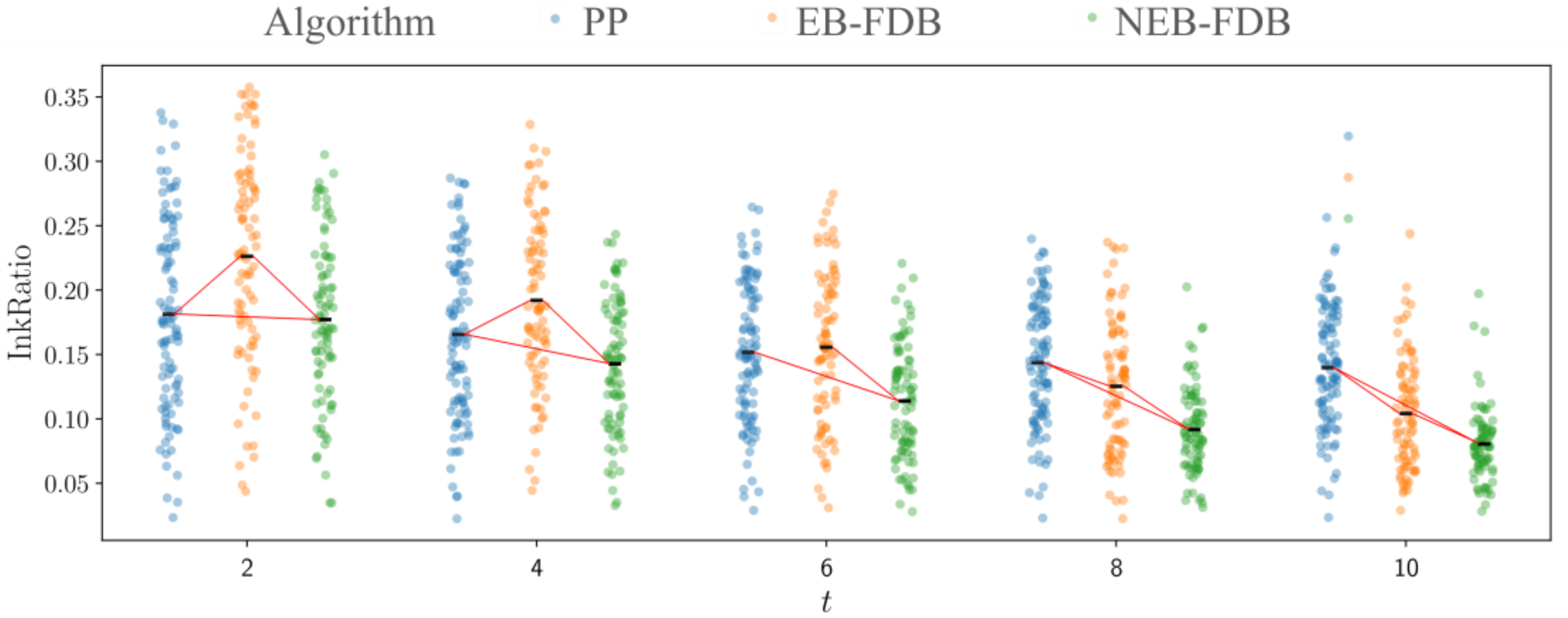
Ink ratio



Experimental Evaluation – Results

Ink ratio

PP-Bundling $<$ EB-FDB $<$ NEB-FDB

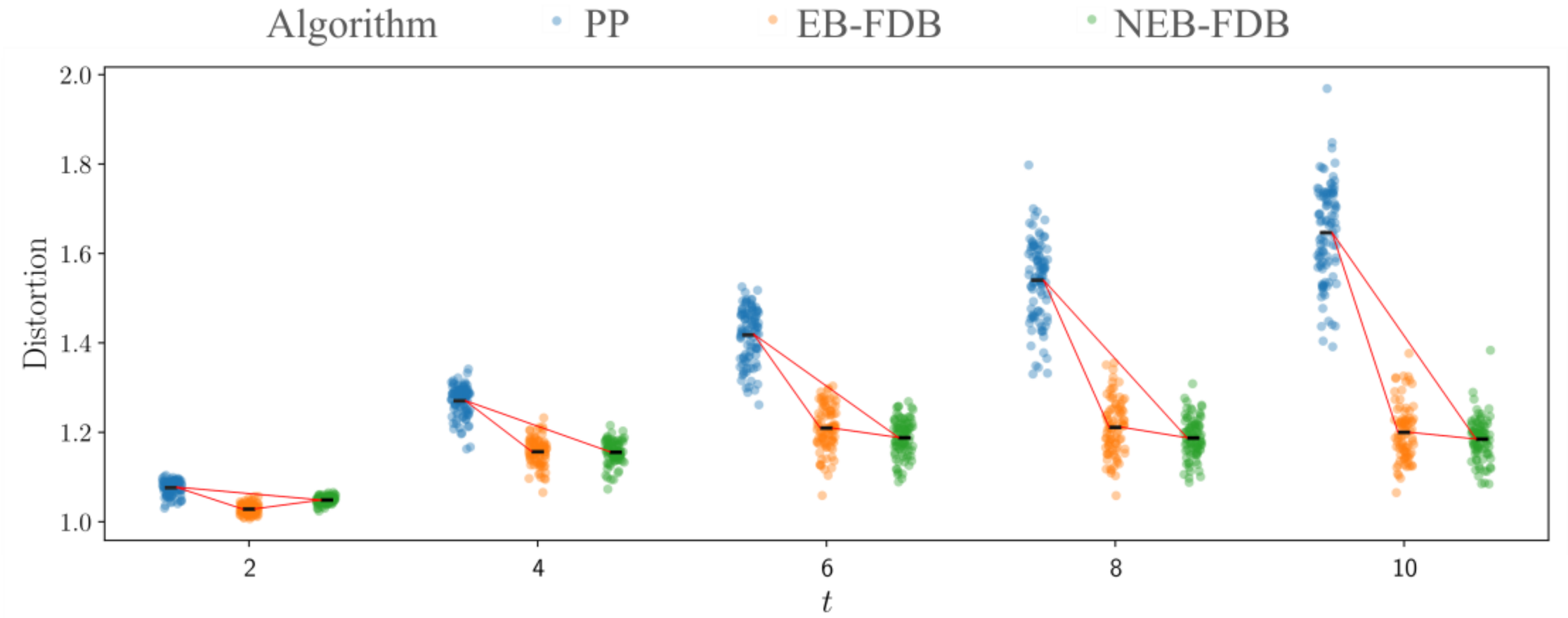


Experimental Evaluation – Results

Ink ratio

Distortion

PP-Bundling $<$ EB-FDB $<$ NEB-FDB



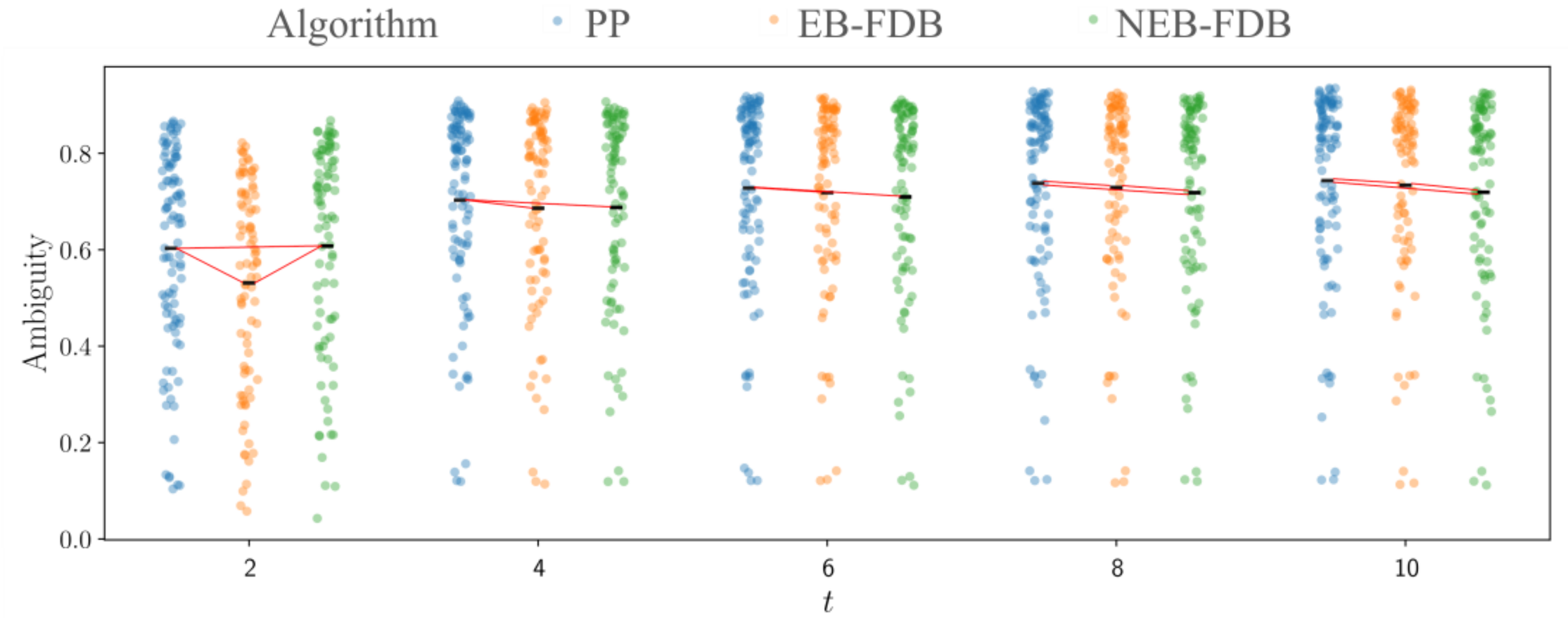
Experimental Evaluation – Results

Ink ratio

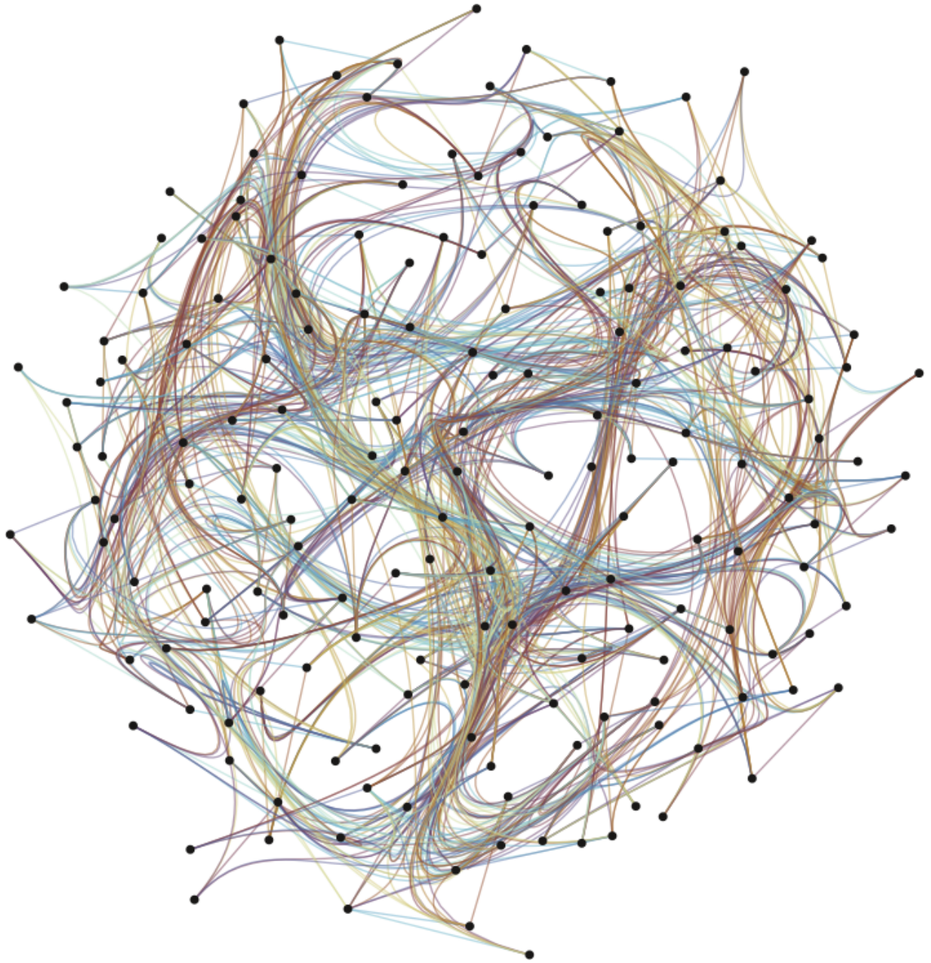
Distortion

Ambiguity

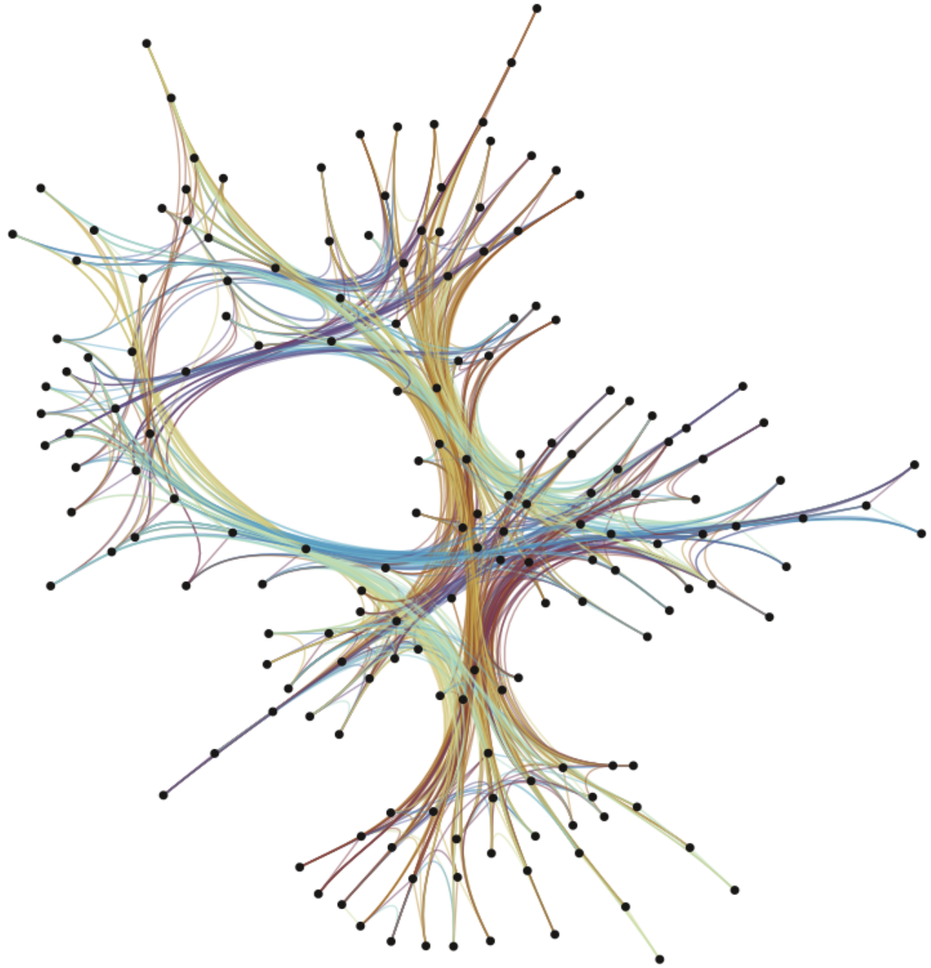
PP-Bundling $<$ EB-FDB $<$ NEB-FDB



Experimental Evaluation – Example

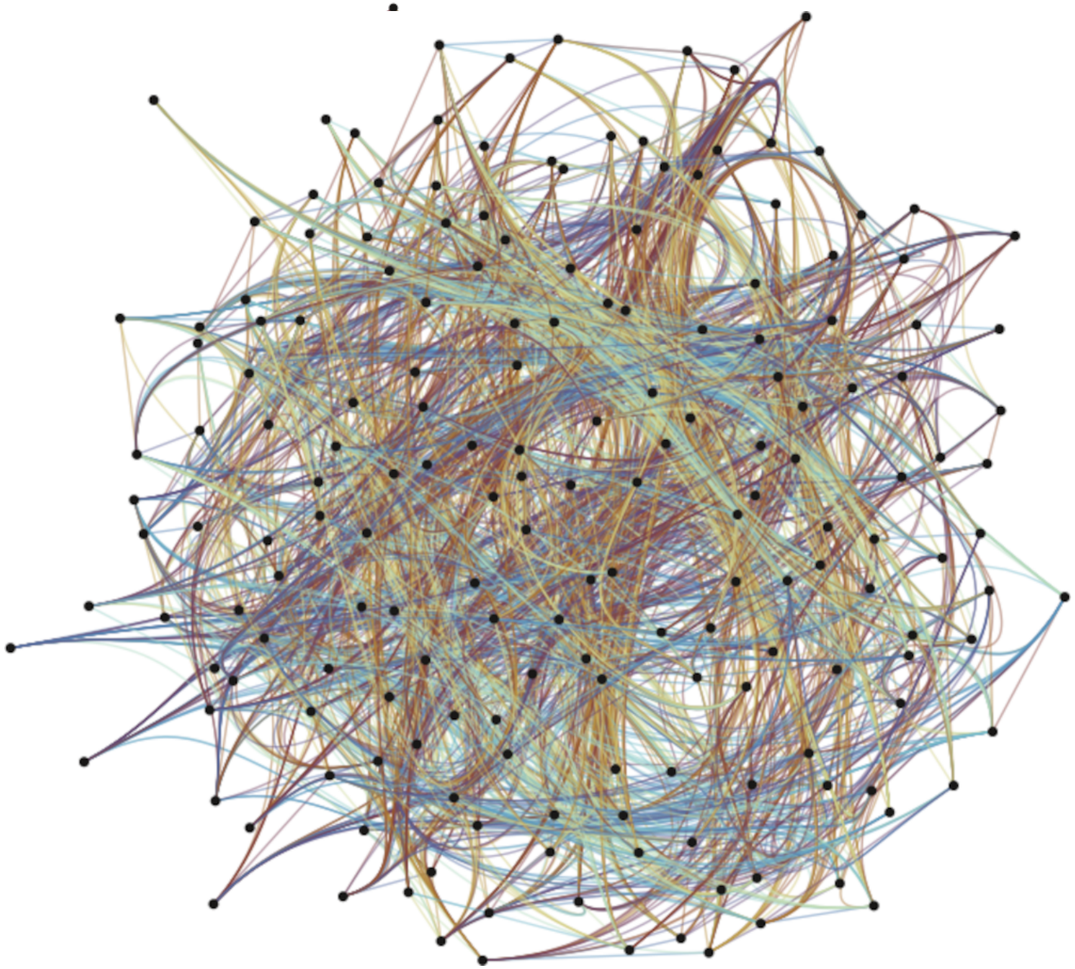


PP-Bundling

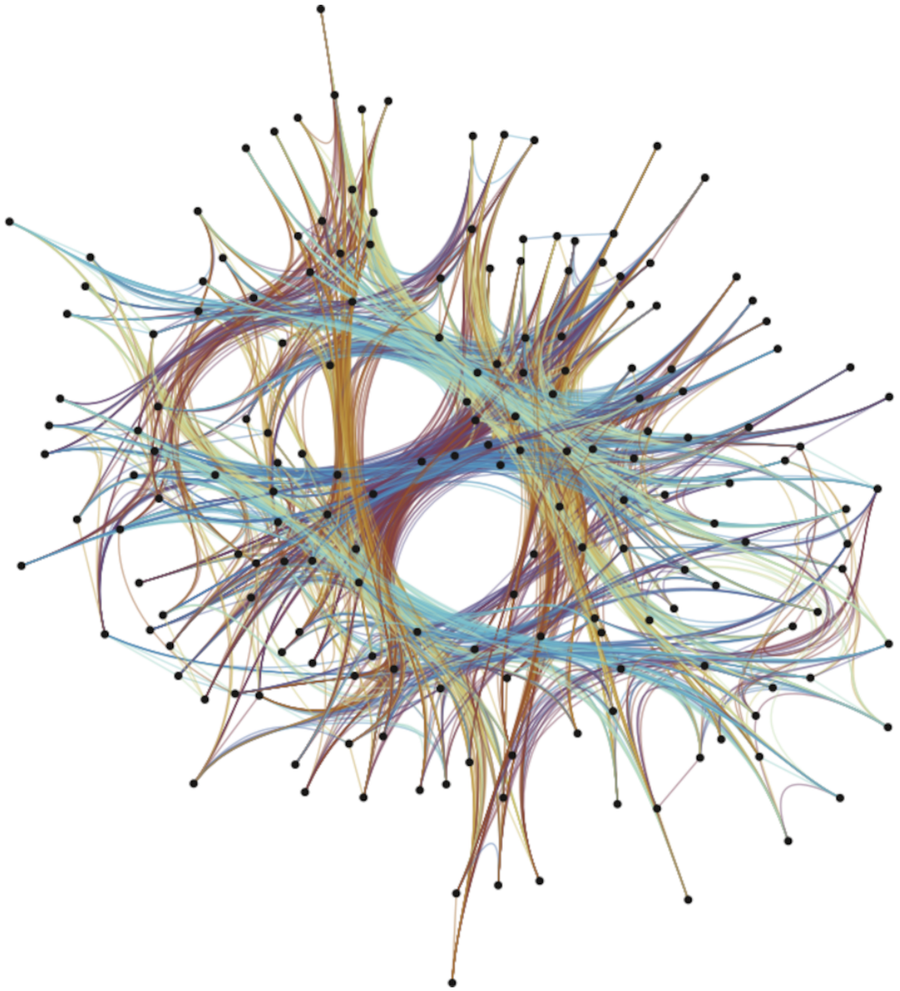


NEB-FDB

Experimental Evaluation – Example

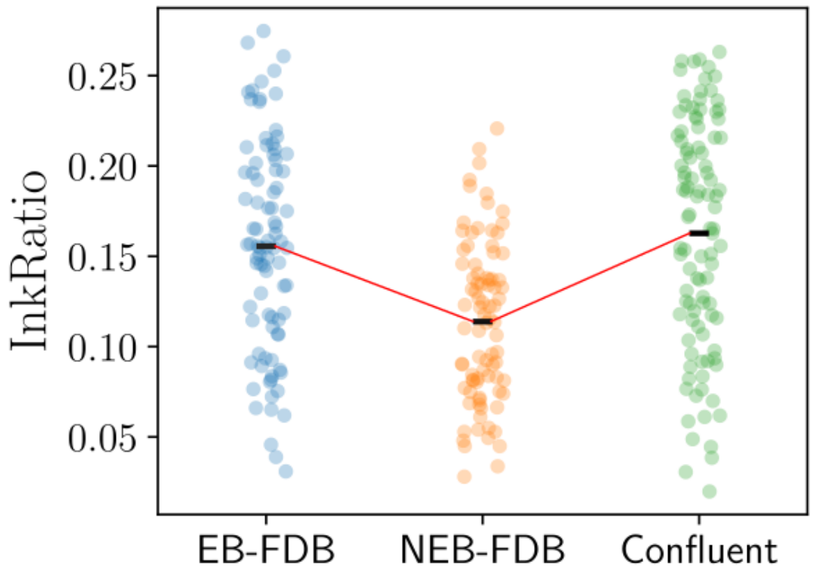


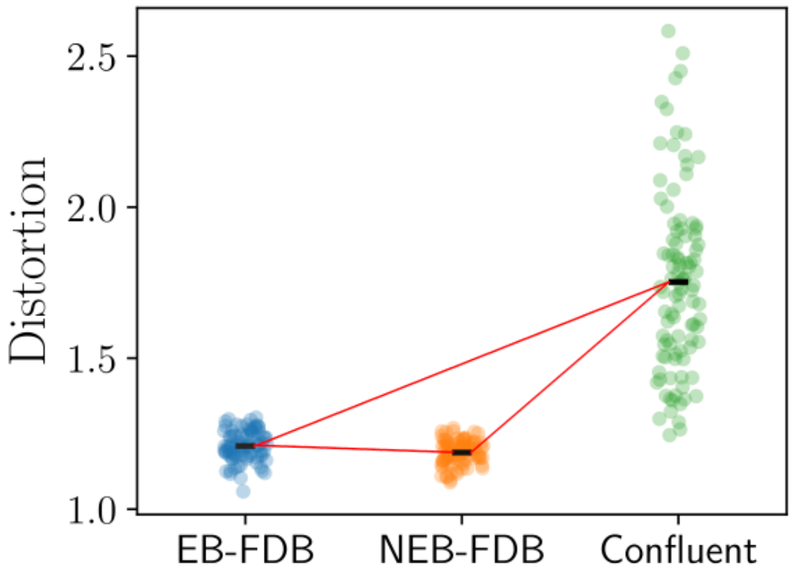
EB-FDB

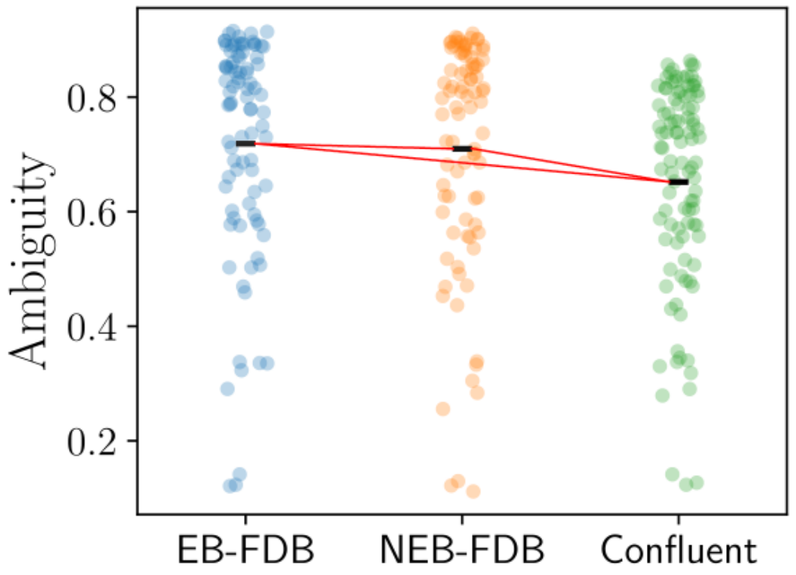


NEB-FDB

Experimental Evaluation – Results





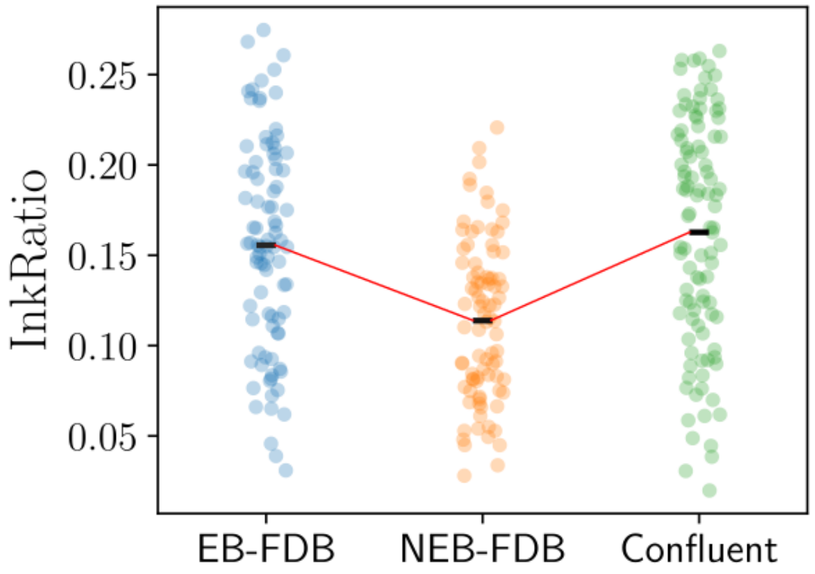


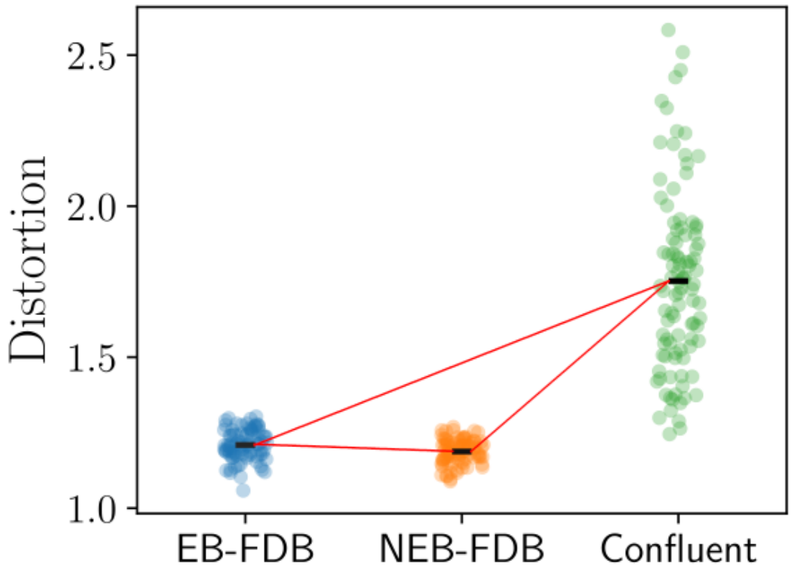
Experimental Evaluation – Results

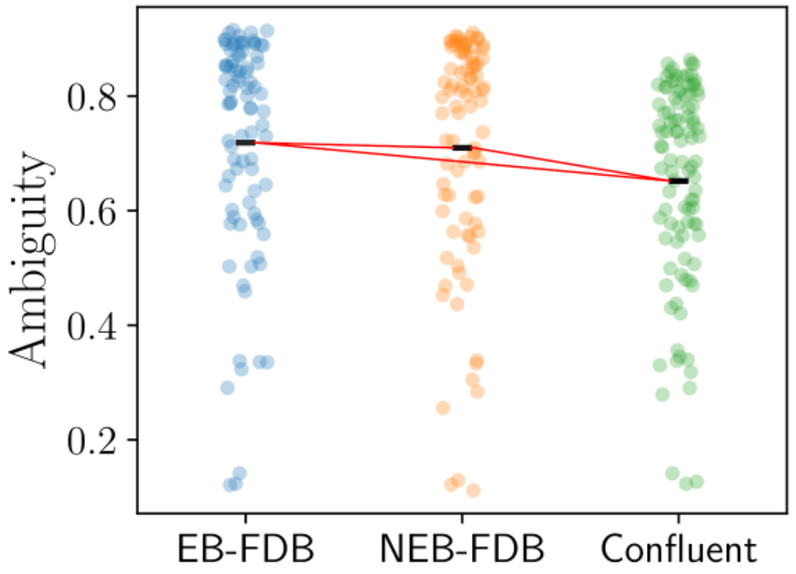
Ink ratio

Distortion

Confluent < EB-FDB < NEB-FDB







Experimental Evaluation – Results

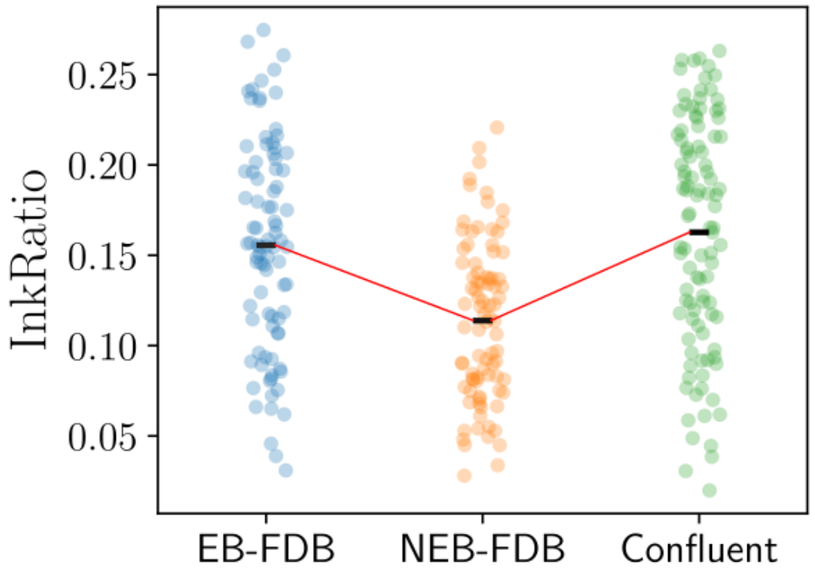
Ink ratio

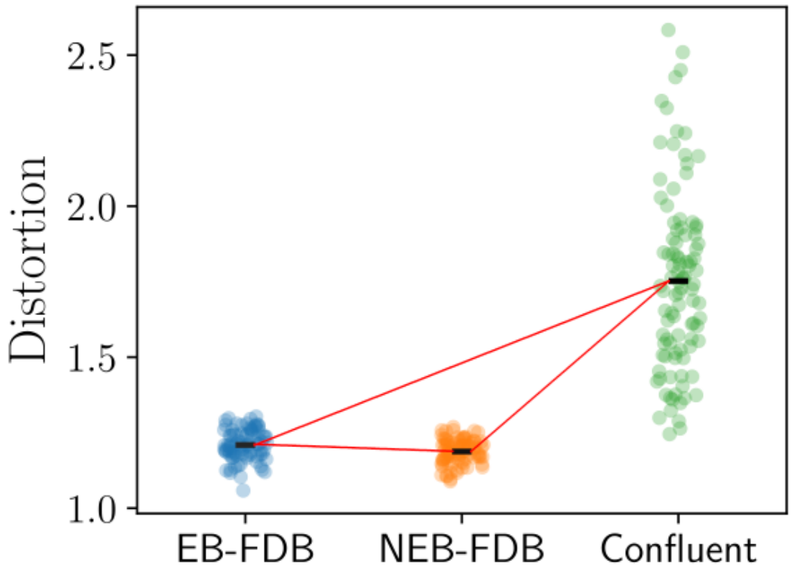
Distortion

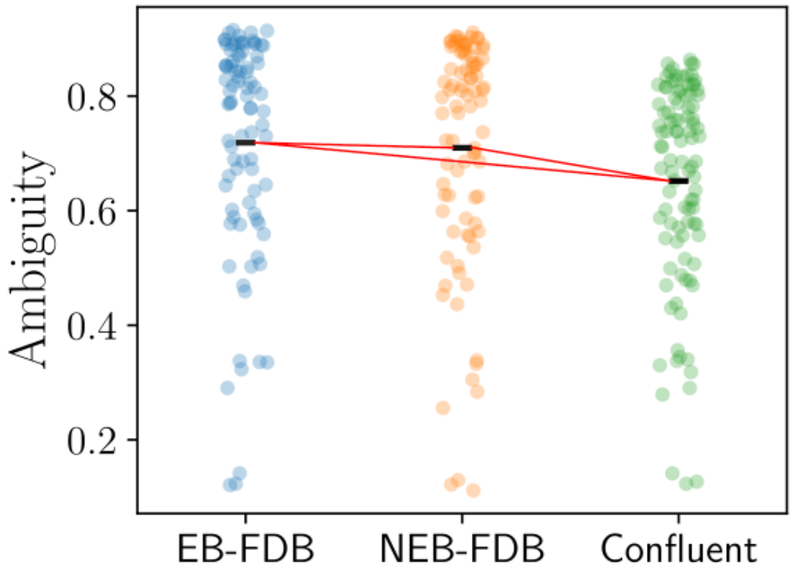
Ambiguity

Confluent < EB-FDB < NEB-FDB

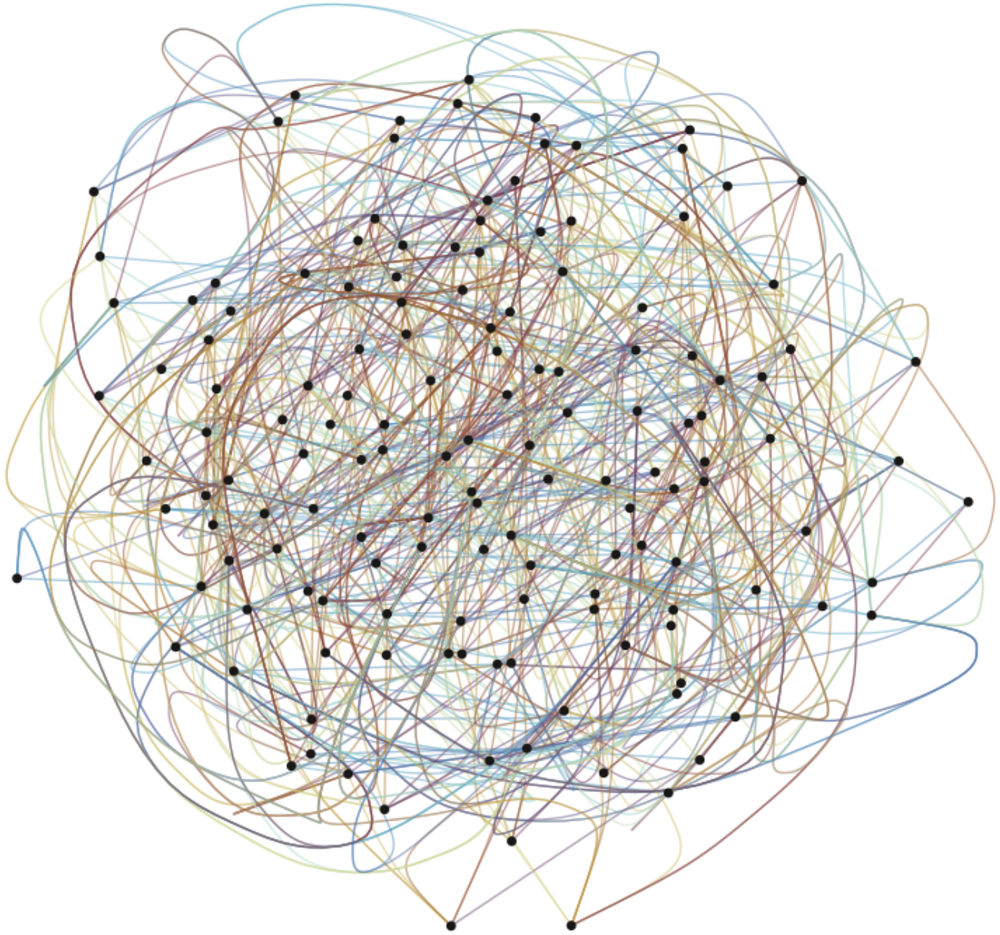
EB-FDB < NEB-FDB < Confluent







Experimental Evaluation – Example

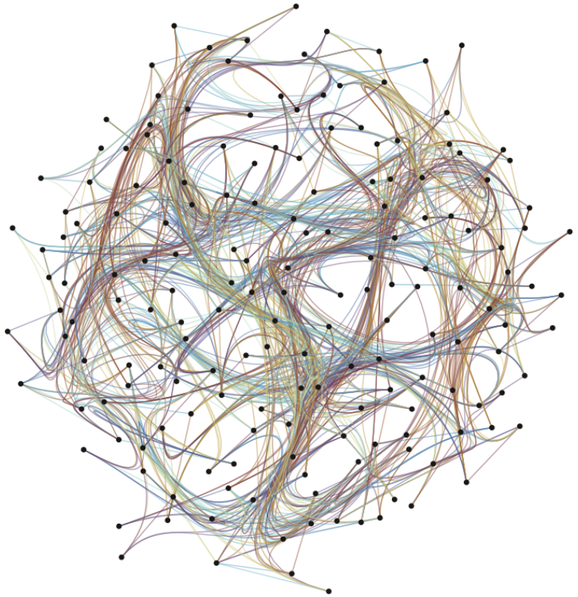


Confluent

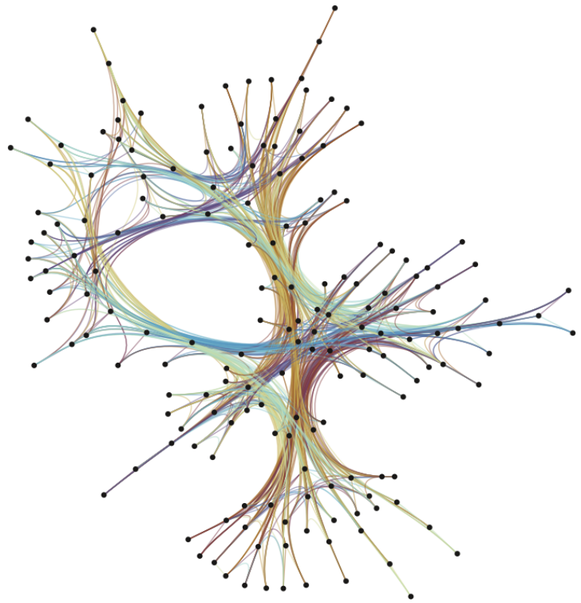
NEB-FDB

Conclusion & Future Work

- FDB significantly enhances the core quality metrics of a bundled drawing
- Edge weights and parameter t influence core quality metrics



- Test different **filter** strategies
- Test different **layout** strategies
- Consider **crossing angle** of (bundled) crossings



Experimental Evaluation – Results

- NEB-FDB filters **sparser** than EB-FDB
- Both exhibit tree structure for $t > 10$